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Computing $[1,2]$-open locating domination number in some families of graphs

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Abstract

The problem of location detection is investigated for many scenarios, such as pointing out the flaws in the multiprocessors, invaders in buildings and facilities, and utilizing wireless sensors networks for the environmental monitoring process. The system or structure can be illustrated as a graph in each of these applications, and sensors strategically placed at a subset of vertices can determine and identify irregularities within the network. The (OLD-set) that is open locating dominating set is a subset of vertices in a graph, such that every vertex within the graph is distinct and non-empty. Let $G = (V, E)$, be the graph, a set $S \subseteq V(G)$ is a $[1,2]$-OLD set if $N(i) \cap S \neq \emptyset$, for some $i \in V(G)$, and $1 \leq |N(i) \cap S| \leq 2$, as well as $N(i) \cap S \neq N(j) \cap S$, for every pair of distinct vertices $i, j \in V(G) \backslash S$. The minimum cardinality of $[1,2]$-OLD set in a graph G is called $[1,2]$-open locating domination number and is denoted by $\gamma^{old}_{[1,2]}$. In this paper, we compute the $[1,2]$-open locating domination number of some families of graphs.

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Keywords: Domination number, Open-locating domination number; $[1,2]$-Open locating domination number; Petersen Graph; Convex Polytopes.

0 Introduction and Preliminaries

Location detection problems have been considered for several applications, including detecting faults in multiprocessors, contaminants in standard utilities, invaders in buildings and amenities, and environmental monitoring employing wireless sensor networks. The system or framework can be modeled as a graph in each of these applications. Sensors strategically placed at a subset of vertices can determine and identify irregularities in the network. An OLD-set(open locating-dominating set) in a graph G is a subset of vertices in that graph such that they have a unique and non-empty set of neighbors in the subset. Sensors positioned at vertices of the OLD-set will detect and identify disruptions in a network in a specific way. Such sensors can be expensive, and therefore it is vital to reduce the size of the OLD-set. Formally the open-locating dominating set S for a graph G

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is the collection of vertices which dominates G, and for any vertices $u, v \in V \backslash S$, the condition $S \cap N(u) \neq S \cap N(v)$ satisfies. The set S will be denoted as OLD-set. The smallest cardinality of such a set is denoted as $\gamma^{old}(G)$. If detector can distinguish an invader at $N(u)$, without the capability of detecting at u, then we examine open-locating dominating set as studied in [20], [21], [6], [19].

A set $S \subseteq V(G)$, of an undirected graph $G = (V, E)$, is a dominating set S if for any vertex that does not relate to S has some neighbors in S. The domination number $\gamma(G)$ is the least size of dominating set in G.

The application prompting this research is modeled using a dominating set of vertices in a graph representing defenders at distinct places in a facility. In this case, we require one defender to get to any un-protective vertex in one step and for a backup defender to be no more than two vertices away.

Definition 1.1 A subset $S \subseteq V$, is a $[g, h]$-set if $i \in V \backslash S$, $g \leq |N(i) \cap S| \leq h$ for positive integers g, and h, the vertex $i \in V \backslash S$ is adjacent to at least g but no more than h vertices in S [7].

Definition 1.2 Let S be a subset of G, is a $[1, 2]$-set, if for some $u \in V \backslash S$ we have $1 \leq |N(u) \cap S| \leq 2$, that is each vertex $u \in V \backslash S$ is adjacent to at least one but no more than two vertices in S. The minimum cardinality of $[1, 2]$-set in G is called $[1, 2]$-domination number and is represented by $\gamma_{[1,2]}(G)$[10], [11].

Definition 1.3 A $S \subseteq V(G)$ is known as total dominating set if for each vertex $v \in V$, $N(v) \cap S \neq \emptyset$. The total domination number is least size of total dominating set S, it is denoted as $\gamma_t(G)$.A total dominating set $S \subseteq V$, is termed as total[1,2]-set if for any vertex $x \in V \backslash S$, $1 \leq |N(x) \cap S| \leq 2$. The total$[1, 2]$-domination of G, is the minimum cardinality of total$[1, 2]$-set, and is denoted as $\gamma_{t[1,2]}(G)$ [14].

If S is a $[1, 2]$-set it is dominating as well, but a dominating may not be a $[1, 2]$-set implies $\gamma_{[1,2]}(G) \geq \gamma(G)$. For any graph G,

$$\gamma(G) \leq \gamma_t(G) \leq \gamma_{t[1,2]}(G),$$

$$\gamma(G) \leq \gamma_{[1,2]}(G) \leq \gamma_{t[1,2]}(G).$$

Definition 1.4 A $S \subseteq V$, is called independent if in a set S, no two vertices are adjacent. The independent domination number $i(G)$ is the cardinality of a minimum independent dominating set in G, that is $\gamma(G) \leq i(G)$. Now with an additional property, a dominating set S is an independent$[1, 2]$-set if $1 \leq |N(j) \cap S| \leq 2$, for every vertex $j \in V \backslash S$. The minimum cardinality of a such a set is called independent$[1, 2]$-number, denoted by $i_{[1,2]}|(G)$. The relation $i(G) \leq i_{[1,2]}(G)$ for every graph admitting an independent$[1, 2]$-set. For further study on this parameter readers can see [9], and [1].

In studies involving safeguard implementations in graphical facilities models or multiprocessor networks, different types of security sets have been studied to precisely locate an "intruder" such as a thief, saboteur, or explosion, or defective processor. It is commonly believed that a system modeling tool located at a vertex v can detect an intruder only if it is at v or a vertex location adjacent to v in a graph $G = (V, E)$. For any vertex $x \in V$, the set $N_G(x) = \{v \in V | (x, y) \in E\}$ is called the *open neighborhood* of x. Moreover, $N_G[x] = N_G(x) \cup \{x\}$ is called the *closed neighborhood* of x.

Motivated by the above notion of $[1, 2]$-domination number, total $[1, 2]$-set, independent $[1, 2]$-set we introduce the $[1, 2]$-open locating domination that is defined in a similar way as above different parameters of domination are defined.

Definition 1.5 A set $S \subseteq V$, is termed as $[1, 2]$-open locating dominating set if for any vertices $u, v \in V \backslash S$, $S \cap N(u) \neq S \cap N(v)$ satisfies, as well as for such vertices $1 \leq |N(u) \cap S| \leq 2$, and $1 \leq |N(v) \cap S| \leq 2$. The minimum cardinality of such a set is called $[1, 2]$-open locating domination number and is denoted as $\gamma^{old}_{[1,2]}(G)$. The relation holds $\gamma^{old}_{[1,2]}(G) \geq \gamma^{old}(G)$.

1 Known results for $\gamma^{old}(G)$

Some of the known results concerning the open-locating domination in a graph G are as follows;

Lemma 1. [20] *For $n \geq 3$, $\gamma^{old}(C_n) = \lceil \frac{2n}{3} \rceil$*

Lemma 2. [20] *For $n \geq 10$, we have;*

$$\gamma^{old}(P_n) = \begin{cases} 4k + r, n = 6k + r & r \in 0, 1, 2, 3, 4; \\ 4k + 4, n = 6k + 5. \end{cases}$$

Lemma 3. [5] *For a complete graph of order n, we have $\gamma^{old}(K_n) = n - 1$.*

Lemma 4. [2] *For $n \geq 10$, and $n \geq 10$, and $n = 10f + h$, and $h \in \{0, 1 \ldots, 9\}$ we have;*

$$\gamma^{old}(P_n^2) \leq \begin{cases} 4f + 1, & \text{if } h \in 0, 1; \\ 4f + 2, & \text{if } h \in 2, 3; \\ 4f + 3, & \text{if } h \in 4, 5; \\ 4f + 4, & \text{if } h \in 6, 7, 8, 9; \end{cases}$$

Lemma 5. [2] *For $n \geq 9$, we have; $\lceil \frac{n}{3} \rceil \leq \gamma^{old}(C_n^2) \leq \lceil \frac{n-2}{2} \rceil + 1$.*

Theorem 1. [5],[20] *Let G be a graph of order n and maximum degree Δ, then;*

$$\gamma^{old}(G) \geq \frac{2n}{\Delta + 1}.$$

Let H be the graph as shown in 1. Now let $S = \{b, d, e, g, h\}$. Now we need to check the intersections, $N(a) \cap S = \{b\}$, $N(b) \cap S = \{d\}$,$N(c) \cap S = \{b, e\}$, $N(d) \cap S = \{b, g\}$, $N(e) \cap S = \{h\}$,

$N(f) \cap S = \{d, g\}$, $N(g) \cap S = \{d, h\}$, $N(h) \cap S = \{e, g\}$, and $N(i) \cap S = \{e, h\}$. It can be clearly seen that all the intersections are non-empty and distinct and as well as intersection of open neighborhood of a vertex with the S contains at least one vertex and no more than two vertices. Thus S is $[1, 2]$-open locating dominating set. Thus $[1, 2] - OLD(H) = |S| = 5$. This also shows that $\gamma^{old}_{[1,2]}(H) = \gamma^{old}(H)$.

Observation 1: A graph G has $[1, 2]$-open locating dominating set if and only if the minimum

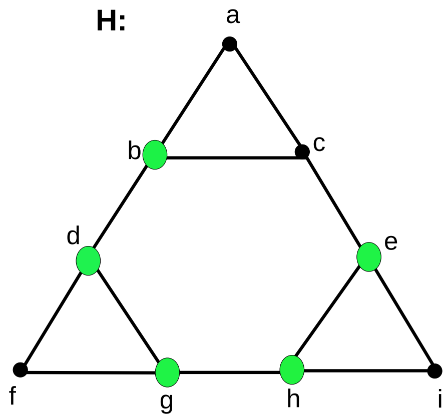

Figure 1: $[1, 2]$-OLD set.

degree of a graph is $\delta \geq 1$, and for some vertices $y \neq z$, we have $N(y) \neq N(z)$, also the intersection of open neighborhood of these vertices with the set $S \subseteq V(G)$, contains at least one and no more than two vertices.

Observation 2: For a graph G, [1, 2]-OLD(G)=2, if and only $G = K_2, K_3$.

Proof: In order to prove let us assume S be an $[1, 2]$-OLD set for a graph G of size 2. Then it is quite clear that the no vertex of S has external private neighbor in $V(G) - S$, and also it satisfies the condition to be $[1, 2]$-OLD set.

In this next section we will find the $[1, 2]$-OLD number of cycle, path, and square of path and cycle graphs, respectively.

2 Upper Bounds for $\gamma^{old}_{[1,2]}$

2.1 Cycle graphs

Let C_n be the cycle graph. The vertices and edges of $V(C_n) = \{a_0, a_1, \dots, a_{n-1}\}$, and $E(C_n) = \{a_0a_1, a_1a_2, \dots, a_{n-1}a_0\}$. The cycle graph is a regular graph of degree 2.

Proposition 1. *For $n \geq 6$, we have*

$$\gamma^{old}_{[1,2]}(C_n) = \begin{cases} \lceil \frac{2n}{3} \rceil, & n \equiv 0, 2 (\mod 3); \\ \lceil \frac{2n}{3} \rceil + 1, & n \equiv 1 (\mod 3). \end{cases}$$

The result for $\gamma^{old}_{[1,2]}(C_n)$ is later used to find the upper bounds for different graphs of convex polytopes.

Proof. In order to prove that we will establish a set S;

$$S = \begin{cases} \{a_{3\ell+1}, a_{3\ell+2} \mid \ell = 0, ..., j-1 & n = 3j; \\ \{a_{3\ell}, a_{3\ell+1} \cup \{a_{3j-1}, a_{3j}\}, \mid \ell = 0, ..., j-1 & n = 3j+1; \\ \{a_{3\ell}, a_{3\ell+1}, \cup \{a_{3j}, a_{3j+1}\} \mid \ell = 0, ..., j-1 & n = 3j+2; \end{cases}$$

The Table1 is presented which clearly shows that, $S \cap N(a) \neq \emptyset$, the intersection with the set S is distinct as well as it can be clearly seen that $1 \leq |N(a) \cap S| \leq 2$. By the above construction one

n	$a \in V$	$S \cap N(a)$	$a \in V$	$S \cap N(a)$
$3j$	$a_{3\ell+1}$	$\{a_{3\ell+2}\}$	$a_{3\ell+2}$	$\{a_{3\ell+1}\}$
	$a_{3\ell+3}(\ell = 0, \ldots, j-2)$	$\{a_{3\ell+2}, a_{3\ell+4}\}(\ell = 0, \ldots, j-2)$	a_0	$\{a_1, a_{3j-1}\}$
$3j+1$	$a_{3\ell+1}(\ell = 0, \ldots, j-2)$	$\{a_{3\ell}\}(\ell = 0, \ldots, j-2)$	$a_{3\ell+2}$	$\{a_{3\ell+1}, a_{3\ell+3}\}$
	$a_{3\ell+3}$	$\{a_{3\ell+4}\}$	a_{3j-2}	$\{a_{3j-3}, a_{3j-1}\}$
	a_{3j}	$\{a_{3j-1}, a_0\}$	a_0	$\{a_1, a_{3j}\}$
$3j+2$	$a_{3\ell+1}$	$\{a_{3\ell}\}$	$a_{3\ell+2}$	$\{a_{3\ell+1}, a_{3\ell+3}\}$
	$a_{3\ell+3}$	$\{a_{3\ell+4}\}$	a_0	$\{a_1, a_{3j+1}\}$
	a_{3j+1}	$\{a_{3j}, a_0\}$		

Table 1: $[1,2]$-OLD vertices in C_n.

can easily notice that S is $[1,2]$-OLD set;

$$\gamma^{old}_{[1,2]}(C_n) \leq \begin{cases} \lceil \frac{2n}{3} \rceil, & n \equiv 0, 2(\mod 3); \\ \lceil \frac{2n}{3} \rceil + 1, & n \equiv 1(\mod 3). \end{cases}$$

Now by lemma 1 we have $\gamma^{old}(C_n) = \lceil \frac{2n}{3} \rceil$. Now from the above facts for $n \equiv 1(mod 3)$, we have $\gamma^{old}_{[1,2]}(C_n) \neq \gamma^{old}(C_n)$. Let us assume on contrary we have $\gamma^{old}_{[1,2]}(C_n) = \gamma^{old}(C_n)$, for $n \equiv 1(mod 3)$. Let us assume set $S = \{a_{3\ell}, a_{3\ell+1}, a_{3j-1} \mid \ell = 0, ..., j-1\}$. We will encounter $a_{3j-3} = a_{3j-1} = \{a_{3j-2}\}$, which is a contradiction as the intersection of the open neighborhood of vertices with the given set S are equivalent. Now we can present an improved result for the $\gamma^{old}(C_n)$.

Theorem 2. *For $n \geq 6$, we have for the graph of C_n;*

$$\gamma^{old}(C_n) = \begin{cases} \lceil \frac{2n}{3} \rceil, & n \equiv 0, 2(\mod 3); \\ \lceil \frac{2n}{3} \rceil + 1, & n \equiv 1(\mod 3). \end{cases}$$

□

2.2 Path Graphs

The $[1,2]$-OLD number of path graph P_n can be computed in a similar way; and from [20] , and by Theorem 1 we have $\gamma^{old}(P_n) = \lceil \frac{2n}{3} \rceil$; so we can write the following result for $[1,2]$-OLD number of P_n as;

Theorem 3. *For $n \geq 6$, we have*

$$\gamma^{old}_{[1,2]}(P_n) = \gamma^{old}(P_n) = \left\lceil \frac{2n}{3} \right\rceil$$

The square of a graph $G = (V, E)$, is the graph $G = (V, E')$, where $E' = E \cup \{xy : d(x, y) = 2\}$. Now we calculate the $[1,2]$-open locating domination number for square of a path and cycle graphs respectively.

2.3 Square of Path Graphs

Now it is not so hard to check that $\gamma^{old}_{[1,2]}(\mathcal{P}^2_5) = 3$, $\gamma^{old}_{[1,2]}(\mathcal{P}^2_6) = +\infty$, and when $n = 7, 8, 9$, we have $\gamma^{old}_{[1,2]}(\mathcal{P}^2_n) = 4$,

Theorem 4. *For $n \geq 10$, we have*

$$\gamma^{old}_{[1,2]}(\mathcal{P}^2_n) \leq \begin{cases} \lceil \frac{2n}{5} \rceil + 1, & \text{when } n \equiv 0,2,3,4,5,6 (mod 10); \\ \lceil \frac{2n}{5} \rceil & \text{when } n \equiv 1,8,9 (mod 10). \end{cases}$$

Proof. In order to prove that we will establish a set S;

$$S = \begin{cases} \{u_{10\ell}, u_{10\ell+2}, u_{10\ell+4}, u_{10\ell+6}\} \cup \{u_{10j-2}\} \mid \ell = 0, ..., j-1 & n = 10j; \\ \{u_{10\ell+1}, u_{10\ell+3}, u_{10\ell+5}, u_{10\ell+7}\} \cup \{u_{10j-1}\} \mid \ell = 0, ..., j-1 & n = 10j+1; \\ \{u_{10\ell}, u_{10\ell+2}, u_{10\ell+4}, u_{10\ell+6}\} \cup \{u_{10j}\}, \{u_{10j+1}\} \mid \ell = 0, ..., j-1 & n = 10j+2; \\ \{u_{10\ell+1}, u_{10\ell+3}, u_{10\ell+5}, u_{10\ell+7}\} \cup \{u_{10j+1}\}, \{u_{10j+2}\} \mid \ell = 0, ..., j-1 & n = 10j+3; \\ \{u_{10\ell+}, u_{10\ell+2}, u_{10\ell+4}, u_{10\ell+6}\} \cup \{u_{10j-2}\}, \{u_{10j+2}\}, \{u_{10j+3}\} \mid \ell = 0, ..., j-1 & n = 10j+4; \\ \{u_{10\ell+1}, u_{10\ell+3}, u_{10\ell+5}, u_{10\ell+7}\} \cup \{u_{10j-1}\}, \{u_{10j+3}\}, \{u_{10j+4}\} \mid \ell = 0, ..., j-1 & n = 10j+5; \\ \{u_{10\ell}, u_{10\ell+2}, u_{10\ell+4}, u_{10\ell+6}\} \cup \{u_{10j-2}\}, \{u_{10j}\}, \{u_{10j+4}\}, \{u_{10j+5}\} \mid \ell = 0, ..., j-1 & n = 10j+6; \\ \{u_{10\ell+1}, u_{10\ell+3}, u_{10\ell+5}, u_{10\ell+7}\} \cup \{u_{10j-1}\}, \{u_{10j+1}\}, \{u_{10j+3}\}, \{u_{10j+5}\} \mid \ell = 0, ..., j-1 & n = 10j+7; \\ \{u_{10\ell}, u_{10\ell+2}, u_{10\ell+4}, u_{10\ell+6}\} \cup \{u_{10j}\}, \{u_{10j+2}\}, \{u_{10j+4}\}, \{u_{10j+6}\} \mid \ell = 0, ..., j-1 & n = 10j+8; \\ \{u_{10\ell+1}, u_{10\ell+3}, u_{10\ell+5}, u_{10\ell+7}\} \cup \{u_{10j+1}\}, \{u_{10j+3}\}, \{u_{10j+5}\}, \{u_{10j+7}\} \mid \ell = 0, ..., j-1 & n = 10j+9. \end{cases}$$

For $2,3,4,5,6,7,8,9 (mod 10)$ only those representation are presented in the next table which differs from the above table.

□

The Tables 2(a),2(b) indicates that there are no vertices in graph with $S \cap N(u) = \infty$, as well as the intersection with the set S is distinct, and $1 \leq |N(u) \cap S| \leq 2$.Finally, computational evidence encourages us to conjecture that Theorem 4 in fact gives the exact values for $\gamma^{old}_{[1,2]}(\mathcal{P}^2_n)$. Note that

n	$u \in V$	$S \cap N(u)$	$u \in V$	$S \cap N(u)$
$10j$	$u_{10\ell}$	$\{u_{10\ell+2}\}$	$u_{10\ell+1}$	$\{u_{10\ell}, u_{10\ell+2}\}$
	$u_{10\ell+2}$	$\{u_{10\ell}, u_{10\ell+4}\}$	$u_{10\ell+3}$	$\{u_{10\ell+2}, u_{10\ell+4}\}$
	$u_{10\ell+4}$	$\{u_{10\ell+2}, u_{10\ell+6}\}$	$u_{10\ell+5}$	$\{u_{10\ell+4}, u_{10\ell+6}\}$
	$u_{10\ell+6} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+4}\} \vert \ell = j-2$	$u_{10\ell+7} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+6}\}\ell = 0, \ldots, j-2$
	$u_{10\ell+8} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+6}, u_{10\ell}\} \vert \ell = 0, \ldots, j-2$	$u_{10\ell+9} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell}\} \vert \ell = 0, \ldots, j-2$
	u_{10j-4}	$\{u_{10j-6}, u_{10j-2}\}$	u_{10j-3}	$\{u_{10j-4}, u_{10j-2}\}$
	u_{10j-2}	$\{u_{10j-4}\}$	u_{10j-1}	$\{u_{10j-2}\}$
$10j+1$	$u_{10\ell}$	$\{u_{10\ell+1}\}$	$u_{10\ell+1}$	$\{u_{10\ell+3}$
	$u_{10\ell+2}$	$\{u_{10\ell+1}, u_{10\ell+3}\}$	$u_{10\ell+3}$	$\{u_{10\ell+1}, u_{10\ell+5}\}$
	$u_{10\ell+4}$	$\{u_{10\ell+3}, u_{10\ell+5}\}$	$u_{10\ell+5}$	$\{u_{10\ell+3}, u_{10\ell+7}\}$
	$u_{10\ell+6}$	$\{u_{10\ell+5}, u_{10\ell+7}\}$	$u_{10\ell+7} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+5}\}\ell = 0, \ldots, j-2$
	$u_{10\ell+8} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+7}\} \vert \ell = 0, \ldots, j-2$	$u_{10\ell+9} \vert \ell = 0, \ldots, j-2$	$\{u_{10\ell+7}, u_{10\ell+11}\} \vert \ell = 0, \ldots, j -$
	u_{10j-3}	$\{u_{10j-5}, u_{10j-1}\}$	u_{10j-2}	$\{u_{10j-3}, u_{10j-1}\}$
	u_{10j-1}	$\{u_{10j-3}\}$	u_{10j}	$\{u_{10j-1}\}$

Table 2(a): $[1,2]$-OLD vertices in $(\mathcal{P}_n^2)$.

every $\gamma^{old}_{[1,2]}(G)$ is an open-locating dominating set $\gamma^{old}(G)$, but the converse is not true for every graph. Next we shown that a complete graph K_n on n vertices do not have open-locating$[1,2]$ dominating set.

Note that every $\gamma^{old}_{[1,2]}(G)$ is an open-locating dominating set $\gamma^{old}(G)$, but the converse is not true for every graph.

2.4 Square Of Cycle Graphs

The squared cycle graph C_n^2 is a regular graph of degree 4. The vertex set is $V(C_n^2) = \{v_0, v_1, \ldots, v_{n-1}\}$. The squared cycle graph is also a special case of Harary graph $H(r, s)$, with $r = 4$. It is straight forward to check that $\gamma^{old}_{[1,2]}(\mathcal{C}_5^2) = +\infty$, $\gamma^{old}_{[1,2]}(\mathcal{C}_6^2) = +\infty$, $\gamma^{old}_{[1,2]}(\mathcal{C}_7^2) = +\infty$, $\gamma^{old}_{[1,2]}(\mathcal{C}_8^2) = +\infty$, $\gamma^{old}_{[1,2]}(\mathcal{C}_9^2) = +\infty$, $\gamma^{old}_{[1,2]}(\mathcal{C}_{10}^2) = 4$, $\gamma^{old}_{[1,2]}(\mathcal{C}_{11}^2) = +\infty$

Theorem 5. *For $\gamma^{old}_{[1,2]}(\mathcal{C}_n^2)$, $n \geq 12$, we have*

$$\gamma^{old}_{[1,2]}(\mathcal{C}_n^2) \leq \begin{cases} \lceil \frac{n-2}{2} \rceil, & \textit{if } n \equiv 0, 1, 2 (mod 4) \ ; \\ +\infty & \textit{otherwise.} \end{cases}$$

Proof. In order to prove that we will establish a set S;

$$S = \begin{cases} \{v_{4\ell+1}, v_{4\ell+3}\} \cup \{v_{4j-3}\} \mid \ell = 0, ..., j-2 & n = 4j; \\ \{v_{4\ell+1}, v_{4\ell+3}\} \cup \{v_{4j-3}\}, \{v_{4j-1}\} \mid \ell = 0, ..., j-2 & n = 4j+1; \\ \{v_{4\ell+1}, v_{4\ell+3}\} \cup \{v_{4j-3}\}, \{v_{4j-1}\} \mid \ell = 0, ..., j-2 & n = 4j+2; \\ \{v_{4\ell+1}, v_{4\ell+3}\} \cup \{v_{4j-3}\}, \{v_{4j-1}\}, \{v_{4j+1}\} \mid \ell = 0, ..., j-2 & n = 4j+3; \end{cases}$$

The Table 3 consider four cases. As it can be clearly seen that for the cases $n = 4j$, $n = 4j+1$, $n = 4j+2$, all the intersection of the vertices in the graph with the set S are non-empty and distinct as well as $1 \leq |N(v) \cap S| \leq 2$. But the for the case $n = 4j+3$, this property is not satisfied. $\square$

n	$u \in V$	$S \cap N(u)$	$u \in V$	$S \cap N(u)$
$10j+2$	$u_{10\ell+6}$	$\{u_{10\ell+4}\}$	$u_{10\ell+7}$	$\{u_{10\ell+6}\}$
	$u_{10\ell+8}$	$\{u_{10\ell+6}, u_{10\ell}\}$	$u_{10\ell+9}\|\ell = 0, \ldots, j-2$	$\{u_{10\ell}\}\|\ell = 0, \ldots, j-2$
	u_{10j-1}	$\{u_{10j}, u_{10j+1}\}$	u_{10j+1}	$\{u_{10j}\}$
$10j+3$	$u_{10\ell+7}$	$\{u_{10\ell+5}\}$	$u_{10\ell+8}$	$\{u_{10\ell+7}\}$
	$u_{10\ell+9}$	$\{u_{10\ell+7}, u_{10\ell+11}\}$	u_{10j}	$\{u_{10j+1}, u_{10j+2}\}$
	u_{10j+1}	$\{u_{10j+2}\}$	u_{10j+2}	$\{u_{10j+1}\}$
$10j+4$	u_{10j}	$\{u_{10j-2}, u_{10j+2}\}$	u_{10j+1}	$\{u_{10j+2}, u_{10j+3}\}$
	u_{10j+2}	$\{u_{10j+3}\}$	u_{10j+3}	$\{u_{10j+2}\}$
$10j+5$	u_{10j+1}	$\{u_{10j-1}, u_{10j+3}\}$	u_{10j+2}	$\{u_{10j+3}, u_{10j+4}\}$
	u_{10j+3}	$\{u_{10j+4}\}$	u_{10j+4}	$\{u_{10j+3}\}$
$10j+6$	$u_{10\ell+8}$	$\{u_{10\ell+6}, u_{10\ell}\}$	u_{10j-1}	$\{u_{10j-2}, u_{10j}\}$
	u_{10j}	$\{u_{10j-2}\}$	u_{10j+1}	$\{u_{10j}\}$
	u_{10j+2}	$\{u_{10j}, u_{10j+4}\}$	u_{10j+3}	$\{u_{10j+4}, u_{10j+5}\}$
	u_{10j+4}	$\{u_{10j+5}\}$	u_{10+5}	$\{u_{10j+4}\}$
$10j+7$	u_{10j}	$\{u_{10j-1}, u_{10j+1}\}$	u_{10j+1}	$\{u_{10j-1}, u_{10j+3}\}$
	u_{10j+2}	$\{u_{10j+1}, u_{10j+3}\}$	u_{10j+3}	$\{u_{10j+1}, u_{10j+5}\}$
	u_{10j+4}	$\{u_{10j+3}, u_{10j+5}\}$	u_{10j+5}	$\{u_{10j+3}\}$
	u_{10j+6}	$\{u_{10j+5}\}$		
$10j+8$	$u_{10\ell+6}$	$\{u_{10\ell+4}\}$	$u_{10\ell+7}$	$\{u_{10\ell+6}\}$
	$u_{10\ell+8}$	$\{u_{10\ell+6}, u_{10\ell}\}$	$u_{10\ell+9}$	$\{u_{10\ell}\}$
	u_{10j}	$\{u_{10j+2}\}$	u_{10j+1}	$\{u_{10j}, u_{10j+2}\}$
	u_{10j+2}	$\{u_{10j}.u_{10j+4}\}$	u_{10j+3}	$\{u_{10j+2}, u_{10j+4}\}$
	u_{10j+4}	$\{u_{10j+2}, u_{10j+6}\}$	u_{10j+5}	$\{u_{10j+4}, u_{10j+6}\}$
	u_{10j+6}	$\{u_{10j+4}\}$	u_{10j+7}	$\{u_{10j+6}\}$
$10j+9$	$u_{10\ell+7}$	$\{u_{10\ell+5}\}$	$u_{10\ell+8}$	$\{u_{10\ell+7}\}$
	$u_{10\ell+9}$	$\{u_{10\ell+7}, u_{10\ell+11}\}$	u_{10j}	$\{u_{10j+1}\}$
	u_{10j+1}	$\{u_{10j+3}\}$	u_{10j+2}	$\{u_{10j+1}, u_{10j+3}\}$
	u_{10j+3}	$\{u_{10j+1}, u_{10j+5}\}$	u_{10j+4}	$\{u_{10j+3}, u_{10j+5}\}$
	u_{10j+5}	$\{u_{10j+3}, u_{10j+7}\}$	u_{10j+6}	$\{u_{10j+5}, u_{10j+7}\}$
	u_{10j+7}	$\{u_{10j+5}\}$	u_{10j+8}	$\{u_{10j+7}\}$

Table 2(b): $[1,2]$-OLD vertices in $(\mathcal{P}_n^2)$.

n	$v \in V$	$S \cap N(v)$	$v \in V$	$S \cap N(v)$
$4j$	v_0	$\{v_1\}$	v_1	$\{v_3\}$
	$v_{4\ell+2}$	$\{v_{4\ell+1}, v_{4\ell+3}\}$	$v_{4\ell+3}$	$\{v_{4\ell+1}, v_{4\ell+5}\}$
	$v_{4\ell+4}$	$\{v_{4\ell+3}, v_{4\ell+5}\}$	$v_{4\ell+5}$	$\{v_{4\ell+3}, (v_{4\ell+7})\|_\ell = 0, \ldots, j-3\}$
	v_{4j-2}	$\{v_{4j-3}\}$	v_{4j-1}	$\{v_{4j-3}, v_1\}$
$4j+1$	v_0	$\{v_1\}$	v_1	$\{v_3\}$
	$v_{4\ell+2}$	$\{v_{4\ell+1}, v_{4\ell+3}\}$	$v_{4\ell+3}$	$\{v_{4\ell+1}, v_{4\ell+5}\}$
	$v_{4\ell+4}$	$\{v_{4\ell+3}, v_{4\ell+5}\}$	$v_{4\ell+5}$	$\{v_{4\ell+3}, (v_{4\ell+7})\}$
	v_{4j-2}	$\{v_{4j-3}, v_{4j-1}\}$	v_{4j-1}	$\{v_{4j-3}\}$
	v_{4j}	$\{v_{4j-1}, v_1\}$		
$4j+2$	v_0	$\{v_1\}$	v_1	$\{v_3\}$
	$v_{4\ell+2}$	$\{v_{4\ell+1}, v_{4\ell+3}\}$	$v_{4\ell+3}$	$\{v_{4\ell+1}, v_{4\ell+5}\}$
	$v_{4\ell+4}$	$\{v_{4\ell+3}, v_{4\ell+5}\}$	$v_{4\ell+5}$	$\{v_{4\ell+3}, (v_{4\ell+7})\}$
	v_{4j-2}	$\{v_{4j-3}, v_{4j-1}\}$	v_{4j-1}	$\{v_{4j-3}\}$
	v_{4j}	$\{v_{4j-1}\}$	v_{4j+1}	$\{v_{4j-1}, v_1\}$
$4j+3$	v_0	$\{v_1, v_{4j+1}, v_{4j+2}\}$	v_1	$\{v_3\}$
	$v_{4\ell+2}$	$\{v_{4\ell+1}, v_{4\ell+3}\}$	$v_{4\ell+3}$	$\{v_{4\ell+1}, v_{4\ell+5}\}$
	$v_{4\ell+4}$	$\{v_{4\ell+3}, v_{4\ell+5}\}$	$v_{4\ell+5}$	$\{v_{4\ell+3}, (v_{4\ell+7})\}$
	v_{4j-2}	$\{v_{4j-3}, v_{4j-1}\}$	v_{4j-1}	$\{v_{4j-3}, v_{4j-1}\}$
	v_{4j}	$\{v_{4j-1}, v_{4j+1}, v_{4j+2}\}$	v_{4j+1}	$\{v_{4j-1}, v_{4j+2}\}$
	v_{4j+2}	$\{v_{4j+1}, v_1\}$		

Table 3: $[1,2]$-OLD vertices in $(\mathcal{C}_n^2)$.

Finally, computational evidence encourages us to conjecture that Theorem 5 in fact gives the exact values for $\gamma^{old}_{[1,2]}(\mathcal{C}^2_n)$.

3 Exact Values

3.1 $[1,2]$-open locating domination number $P(n,k)$

The introduction of generalized Petersen graphs was done by Watkins[24]. The $P(n,k)$, where $n \geq 3$, and $1 \leq k \leq \lfloor \frac{n-1}{2} \rfloor$, is the cubic graph consist of vertices and edges.

$$\mathcal{V}(P(n,k) = \{a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_{n-1}\}$$

$$\mathcal{E}(P(n,k) = \{a_i a_{i+k}, b_i b_i, b_i b_{i+1} | i = 0, 1, \ldots, n-1\}$$

Recently [8] calculated the $\gamma_{[1,2]}(P(n,k))|_{k\in 1,2,3}$, and proved that $\gamma_{[1,2]}(P(n,k)) = \gamma(P(n,k))_{k\in 1,3}$, and also $\gamma_{[1,2]}(P(n,k)) \neq \gamma(P(n,k))_{k=2}$, except for $n = 6, 7, 9, 12$. So motivated by this we calculated $\gamma^{old}_{[1,2]}(P(n,k))$, as shown in the following theorem.

Theorem 6. *For $n \geq 6$, the $\gamma^{old}_{[1,2]}(P(n,k))$ is given as;*

$$\gamma^{old}_{[1,2]}(P(n,k)) = n$$

Proof. Let us construct a set S of $P(n,k)$, now $S = \{b_\ell | \ell = 0, 1, \ldots, n-1\}$. Now the following table is presented; The table clearly there are no vertices in a graph with the empty set and intersection

v	$S \cap N(v)$
b_0	$\{b_1, b_{n-1}\}$
b_1	$\{b_0, b_2\}$
b_ℓ	$\{b_{\ell-1}, b_{\ell+1}|_{\ell=2,3,\ldots,n-1}\}$
a_ℓ	$\{b_\ell\}|_{\ell=0,1,\ldots,n-1}\}$

Table 4: $[1,2]$-OLD vertices in $P(n,k)$.

of the vertices of a graph with the set S are distinct as well, and the condition $1 \leq |S \cap N(v)| \leq 2$ implying that,

$$\gamma^{old}_{[1,2]}(P(n,k)) \leq n$$

Now on the other hand we have $\gamma^{old}_{[1,2]}(G) \geq \gamma^{old}(G)$, and we know the fact that generalized Petersen graph is a graph with each vertex of degree 3, with $2n$ vertices so by using Theorem1, the open-locating domination number can be calculated. Combining all these we can say that;

$$\gamma^{old}_{[1,2]}(P(n,k)) = \gamma^{old}(P(n,k)) = n$$

□

3.2 $[1,2]$-open locating domination number of Convex Polytopes

The graph of convex polytope D_n, consist of $2n$ 5-sided faces and a pair of n-sided faces, as shown in Figure 2. The open locating domination number of D_n was considered in [19]. Motivated by this, in this paper, we found $[1,2]$-open locating domination number of already studied classes of convex polytopes. We gave the exact values of $[1,2]$-open locating domination number, of three further variations of the D_n graph. These are R_n,[16] in which they studied the vertex-magic total labelling of R_n. Imran et al. [13] studied the minimum metric dimension problem. The graph of H_n and H'_n, is studied in [17], where the binary locating dominating number is calculated. By using Theorem 1, for open-locating dominating number we see that for these particular families of convex polytopes, $\gamma^{old}_{[1,2]}(G) = \gamma^{old}(G)$. For other families of convex polytopes, their upper bounds are presented. The

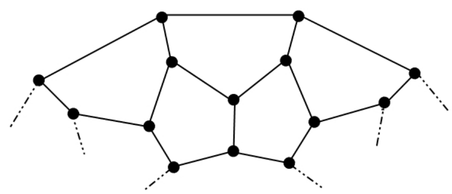

Figure 2: The graph of convex polytope D_n.

authors in [19] found the open locating dominating number of D_n, and proved that,

Theorem 7. [19]*For $n \geq 6$, the open-locating domination number is;*

$$\gamma^{old}(D_n) = 2n$$

3.3 Convex polytope R_n

The graph of convex polytope R_n constitutes of $2n$ 5-sided faces, n6-sided faces, and n-sided faces, as studied in Miller et al. [16]. The vertex and edge set comprises of;

$$V(R_n) = \{a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell \mid \ell = 0, \dots, n-1\};$$

$$E(R_n) = \{a_\ell a_{\ell+1}, a_\ell b_\ell, b_\ell c_\ell, b_\ell c_{\ell-1}, c_\ell d_\ell,$$
$$d_\ell e_\ell, d_\ell e_{\ell+1}, e_\ell f_\ell, f_\ell f_{\ell+1} \mid \ell = 0, \dots, n-1\}.$$

Theorem 8. *For $n \geq 6$, we have;*

$$\gamma^{old}_{[1,2]}(\mathcal{R}_n) = 3n$$

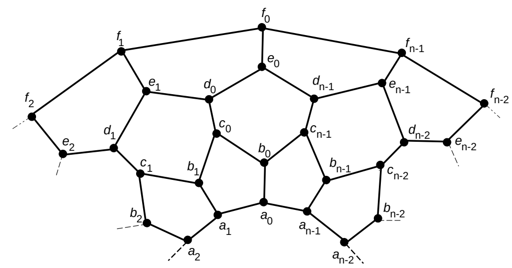

Figure 3: Convex polytope R_n.

v	$S \cap N(v)$
a_ℓ	$\{a_{\ell-1}, a_{\ell+1}\}$
b_ℓ	$\{a_\ell\}$
c_ℓ	$\{d_\ell\}$
d_ℓ	$\{e_\ell, e_{\ell+1}\}$
e_ℓ	$\{d_{\ell-1}, d_\ell\}$
f_ℓ	$\{e_\ell\}$

Table 5: $[1,2]$-OLD vertices in $\mathcal{R}_n$.

Proof. In order to prove that let use consider $S = \{a_\ell, d_\ell, e_\ell | \ell = 0, 1, \ldots, n-1\}$. Now a table is given; It can be seen from above that these intersections are non-empty as well as distinct. The other condition also satisfies as we can see that for some vertex which is in set S, as well as not in S, the intersection of open neighborhood of vertices with the set S comprises of at least one and no more than two vertices. This fact is also clear from the table. So S is $[1,2]$-OLD set f R_n. So $|S| = 3n$, $\gamma^{old}_{[1,2]}(R_n) \leq 3n$. On the other hand $\gamma^{old}_{[1,2]}(R_n) \geq \gamma^{old}(R_n)$ and by Theorem 1 $\gamma^{old}(R_n) \geq \lceil \frac{2.6n}{3+1} \rceil = 3n$. So from all the above facts; $\gamma^{old}_{[1,2]}(R_n) = \gamma^{old}(R_n) = 3n$. □

3.4 Convex polytope H_n

The graph of convex polytope H_n, as studied in[17] where the binary locating domination number is considered. For the sake of simplicity we present vertex and edge set of $\mathcal{H}_n$ as,

$$V(H_n) = \{a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell \mid \ell = 0, \ldots, n-1\};$$

$$\begin{aligned} E(H_n) \;\; = \;\; & \{a_\ell a_{\ell+1}, a_\ell b_\ell, b_\ell c_\ell, b_\ell c_{\ell-1}, \\ & c_\ell d_\ell, d_\ell e_\ell, d_\ell e_{\ell+1}, e_\ell f_\ell, f_\ell g_\ell, \\ & f_\ell g_{\ell-1}, g_\ell h_\ell, h_\ell h_{\ell+1} \mid \ell = 0, 1, \ldots, n-1.\} \end{aligned}$$

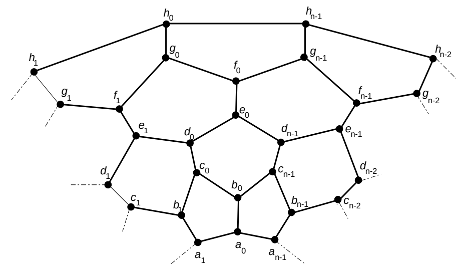

Figure 4: Convex polytope H_n.

Theorem 9. *For $n \geq 8$, we have;*

$$\gamma^{old}_{[1,2]}(H_n) = 4n$$

Proof. In order to prove that let use consider $S = \{a_\ell, d_\ell, e_\ell, h_\ell | \ell = 0, 1, \ldots, n-1\}$. Now a table is shown; It can be seen from the table that these intersections are non-empty as well as distinct.

v	$S \cap N(v)$
a_ℓ	$\{a_{\ell-1}, a_{\ell+1}\}$
b_ℓ	$\{a_\ell\}$
c_ℓ	$\{d_\ell\}$
d_ℓ	$\{e_\ell, e_{\ell+1}\}$
e_ℓ	$\{d_{\ell-1}, d_\ell\}$
f_ℓ	$\{e_\ell\}$
g_ℓ	$\{h_\ell\}$
h_ℓ	$\{h_{\ell+1}, h_{\ell-1}\}$

Table 6: $[1, 2]$-OLD vertices in H_n.

The other condition also satisfies as we can see that for some vertex which is in set S, as well as not in S, the intersection of open neighborhood with the set S comprises of at least one and no more than two vertices.This fact is also clear from the table. So S is $[1, 2]$-open locating dominating set of H_n. So $|S| = 4n$, $\gamma^{old}_{[1,2]}(H_n) \leq 4n$. On the other hand $\gamma^{old}_{[1,2]}(H_n) \geq \gamma^{old}(H_n)$ and by Theorem 1 $\gamma^{old}(H_n) \geq \lceil \frac{2.8n}{3+1} \rceil = 4n$. From all these above facts; $\gamma^{old}_{[1,2]}(H_n) = \gamma^{old}(H_n) = 4n$. □

3.5 Convex polytope H'_n

The graph of convex polytope H'_n, as studied in[17] where the binary locating domination number is considered. For the sake of simplicity we present vertex and edge set of H'_n as,

$$V(H'_n) = \{a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell,$$
$$g_\ell, h_\ell, i_\ell, j_\ell, k_\ell, l_\ell | \ell = 0, \ldots, n-1\}$$

and the edge set

$$\begin{aligned} E(H'_n) &= \{a_\ell a_{\ell+1}, a_\ell b_\ell, b_\ell c_\ell, b_\ell c_{\ell-1}, c_\ell d_\ell, d_\ell e_\ell, \\ &\quad d_\ell e_{\ell+1}, e_\ell f_\ell, f_\ell g_\ell, f_\ell g_{\ell-1}, g_\ell h_\ell, h_\ell i_\ell, \\ &\quad h_\ell i_{\ell+1}, i_\ell j_\ell, j_\ell k_\ell, j_\ell k_{\ell-1}, \\ &\quad k_\ell l_\ell, l_\ell l_{\ell+1} \mid \ell = 0, 1, \ldots, n-1\}. \end{aligned}$$

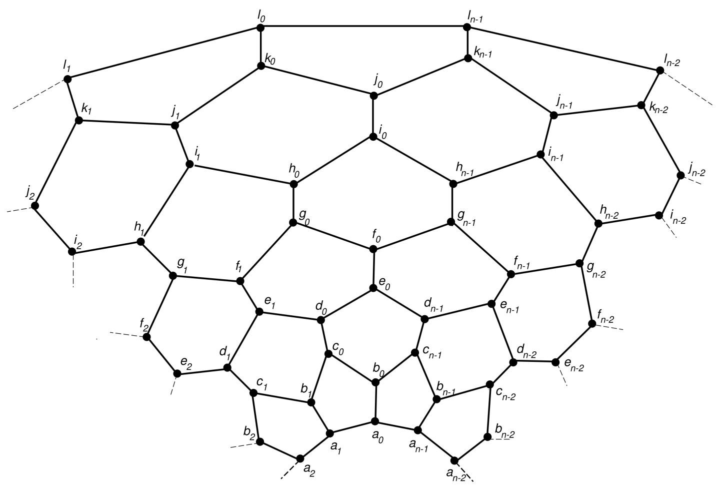

Figure 5: Convex polytope H'_n.

Theorem 10. *For $n \geq 8$, we have;*

$$\gamma^{old}_{[1,2]}(H'_n) = 6n$$

Proof. In order to prove that let use consider $S = \{a_\ell, d_\ell, e_\ell, h_\ell, i_\ell, l_\ell | \ell = 0, 1, \ldots, n-1\}$. Now we present a table; It can be seen from above that these intersections are non-empty as well as distinct. The other condition also satisfies as we can see that for some vertex which is in set S, as well as not in S, the intersection of open neighborhood with the set S comprises of at least one and no more than two vertices.This fact is also clear from the table. So S is $[1,2]$-open locating dominating set of (H'_n). So $|S| = 6n$, $\gamma^{old}_{[1,2]}(H'_n) \leq 6n$. On the other hand $\gamma^{old}_{[1,2]}(H'_n) \geq \gamma^{old}(H'_n)$ and by Theorem 1 $\gamma^{old}(H'_n) \geq \lceil \frac{2.12n}{3+1} \rceil = 6n$. Combining all these we get; $\gamma^{old}_{[1,2]}(H'_n) = \gamma^{old}(H'_n) = 6n$. □

We studied three families of convex polytopes which are actually the extension of the graph of convex polytope D_n, and we conjecture that these are the exact values of $[1,2]$-open locating domination number.

v	$S \cap N(v)$
a_ℓ	$\{a_{\ell-1}, a_{\ell+1}\}$
b_ℓ	$\{a_\ell\}$
c_ℓ	$\{d_\ell\}$
d_ℓ	$\{e_\ell, e_{\ell+1}\}$
e_ℓ	$\{d_{\ell-1}, d_\ell\}$
f_ℓ	$\{e_\ell\}$
g_ℓ	$\{h_\ell\}$
h_ℓ	$\{i_\ell, i_{\ell+1}\}$
i_ℓ	$\{h_{\ell-1}, h_\ell\}$
j_ℓ	$\{i_\ell\}$
k_ℓ	$\{l_\ell\}$
l_ℓ	$\{l_{\ell-1}, l_{\ell+1}\}$

Table 7: $[1, 2]$-OLD vertices in $\mathcal{H}'_n$.

4 Conclusion

In this paper, we initiated the study of $[1, 2]$-open locating domination number of graphs. The study of $[1, 2]$-set in graphs, $[1, 2]$-domination number, Total$[1, 2]$-domination, as well other studies related to these topics is the cause of this study. We calculated the $[1, 2]$-open location domination number of cycle, sqaure of cycle and path respectively as well as studied the generalized petersen graph $G(P(n, k))$, and the graphs of convex polytopes, which are an essential class of graphs from both geometric and combinatorial viewpoint. The further study can be directed towards finding $[1, 2]$-open locating domination of other well-considered graphs. It would be interesting to find $[1, 2]$-set in the trees and finding the bounds and characterizing the trees that possess $[1, 2]$-sets. Recently different domination parameters have been studied for the classes of convex polytopes, so further research can be carried out for those parameters which exhibit the exact values among these families of graphs.

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