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Numerical Investigation of Suction/Injuction on Triple Diffusive MHD Casson Fluid Flow Over a Vertical Stretching Surface

Hafiz Muhammad Irfan[1], **Anum Khan**[2†]

Abstract The article examines the characteristics of heat transfer for steady two-dimensional MHD Casson fluid with shear thickening properties in the presence of variable heat source for a vertical stretching sheet. The study considers the thermal radiation's impact and velocity slip, and uses similarity transformations to convert the governing PDEs and corresponding boundary conditions into ODEs. Numerical solutions are obtained using shooting and R. K-4 methods, and the results presented for both injection and suction cases. The article examines the influence of various neutral parameters on the distribution of temperature, skin friction coefficient, velocity, and local Nusselt number, and presents the findings using tables and graphs. The study reveals that an increase in the Biot number and thermal radiation parameter results in an enhancement of the temperature distribution, while an increase in the suction/injection parameter causes a reduction in the velocity and temperature distributions for both injection and suction cases.
Keywords: Vertical sheet , Magnetohydrodynamics, Suction/Injunction, Casson fluid.
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1. Introduction

The transfer of flow and thermal power in both Newtonian and non-Newtonian liquids holds great importance in various fields such as industry, research, and de-

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[†]Corresponding author: anum.khan@superior.edu.pk
[1] School education department, Punjab, Pakistan
[2] Department of Mathematics, Superior University, Lahore 54000, Pakistan.

sign. According to Newton's viscosity law, there is a direct correlation between the shear stress and the rates of stress. A liquid that complies with Newton's regulation of activity is called Newtonian fluid. Different types of non-Newtonian liquids are classified, including Maxwell fluid, Casson, Williamson, viscoelastic fluid, pair tension fluid, minipolar liquid, power-law flow, and Jeffrey liquid, among others. It is made of plastic and comes in Casson's liquid modal. Attributes such as high shear viscosity and return tension are evident. Khan et al. (Khan et al., 2015) investigated the impacts of viscous dilution on convective boundary conditions in both moving and fixed border scenarios for Casson fluid. Researchers Ramzan et. al., [(Gul et al., 2020)] Chung, Kadry, and Chu studied the thick dissipation of Casson nano fluid under thermal limits. In the case of Mahanta and Shaw. This study examined the blended convection Casson stagnation factor circulation past convective border problems Mahanta and Shaw [(Mahanta and Shaw, 2015),]. Using MHD Casson fluid, Bortteir, Seini, [(Bortteir et al., 2015),] evaluated the fluid's reaction with an upright permeable surface. //

A shear-thinning non-Newtonian liquid's circulation is defined by the Williamson liquid design. Visco-inelastic liquid is Williamson. There are two types of visco-inelastic liquids: thick and flexible. These equations describe the rheological properties of fluids using Navier-Stokes equations. As the liquid has molecular structure, a real fluid has a maximum thickness and minimum viscosity. Pseudo plastic liquids were studied by Williamson [(Williamson, 1929)]. "Williamson liquid warmth transfer" is the concept proposed by Nadeem and Hussain [(Nadeem and Hussain, 2014)]. A liquid that complies with Newton's regulation of activity is called Newtonian fluid. Different types of non-Newtonian liquids are classified, including Maxwell fluid, Casson, Williamson, viscoelastic fluid, pair tension fluid, minipolar liquid, power-law flow, and Jeffrey liquid, among others. Different surface conditions, such as a warmed surface, stretched surface, and significantly stretched surface, have been examined by the researchers. They looked at the effect of MHD on non-Newtonian Williamson-nano liquid in porous medium with chain reaction over a very hot surface area transfer condition Krishnamurthy et. al.,[(Krishnamurthy et al., 2016)]. This paper was published in 2007 by Dapra and Scarpi [(Dapra and Scarpi, 2007)]. A perturbation method was used to fix the problem. According to Alsaedi et al.,[(Hayat et al., 2016)]

Williamson fluid circulation with thermal radiative and Ohmic dissipation over hydro magnetic border layer circulation has been studied. The liquid version of Maxwell is most useful for polymers with low molecular weight. In order to determine the tension relaxation, Maxwell model is used. Nano-Maxwell stagnancy factor cir- culation liquid towards permeable surface was studied by Ramesh and coworkers Ramesh, Gireesha, Hayat, and Alsaedi,Ramesh et. al., [(Ramesh et al., 2016)]. Muhammad Ibrahim Ibrahim [(Ibrahim, 2016)] studied the warm transfer of UCM-Nanofluid point flow and past a convective sheet in MHD stagnation. Unsteady MHD UCM liquid circulation with nonlinear chemical process towards stretching surface area was discussed by Kameswaran, Sui, Zheng, and Zhang in 2016 Kameswaran et. al., [(Sui et al., 2016)]. It was discovered that UCM nano liquid circulation and thermal energy as well as mass transfer can be achieved by using Cattaneoe and Christov double diffusion in the direction of a stretching sheet with slip velocity by Zhang and colleagues Zhang et. al., [(Sui et al., 2016)] Researchers looked at the effects of a transverse electromagnetic field and radiant heat on energy and warmth transfer behaviour for Casson, Maxwell, Williamson, and Newtonian liquids flowing past a suction/injection surface in this research study. It has also been used in a wide range of engineering tasks, such as thrust bearing and radial diffuser designs, as well as thermal oil recovery. As part of chemical procedures, vacuum is used to remove catalysts. In order to fix the problem, we applied a perturb. method. Williamson fluid circulation with thermal radiative and Ohmic dissipation across hydrmagnetic border layer circulation has been examined, according to Alsaedi et. al.,[(Hayat et al., 2016)]. When it comes to polymers with low molecular weight, the liquid form of Maxwell is the most useful. Maxwell's model is used to determine the tension relaxation in order to calculate the tension. RAMESH, GIREESHA, HAYAT, AND ALSAEDI Ramesh et. al., [(Ramesh et al., 2016)] examined nano-Maxwell stagnancy factor cir- culation liquid approaching permeable surface. For example, (Mukhopadhyay and also Vajravelu [(Mukhopadhyay, 2011)] explain that blowing can be used to add catalysts, which cool down the surface area, protect against corrosion or scaling, and reduce drag. Physicists and designers alike use radiative effects in their work. Warm-transfer radiations have a significant impact on various circulations, and they predate both modern technology and heat-producing processes. Radiation's effects on the boundary layer, on the other hand, are largely unknown. When it comes to the polymer processing industry, where the quality of the final product is largely determined by heat management aspects (Mukhopadhyay, Bhattacharyya, (Mukhopadhyay et al. [(Mukhopadhyay et al., 2011)], radiant heat results may play a crucial role in controlling warm transfer. Radiative heat transfer has many application from an upright wall surface to conductive grey liquids (Mukhopadhyay and also Gorla [(Mukhopadhyay et al., 2011)]. With this existing research project, the goal is to extend the flow and evaluate the heat transfer during the boundary layer circulation of a Casson fluid over a substantially extending sheet. On the other hand, radiant heat and suction/blowing have mixed effects that are explored. Relative changes allow us to derive both a 3rd and a 2nd order regular differential formula for the momentum equation and the warmth equation. As much as possible, numerical calculations were carried out for different values of dimensionless parameters. As a matter of fact, the outcomes have been hotly debated. Many other variables such as suction or fired at the wall have a significant impact on the flow area and also heat transmission. Last but not least, the evaluation of skin rubbing, which is crucial from a commercial perspective, is also discussed.

2. Mathematical model

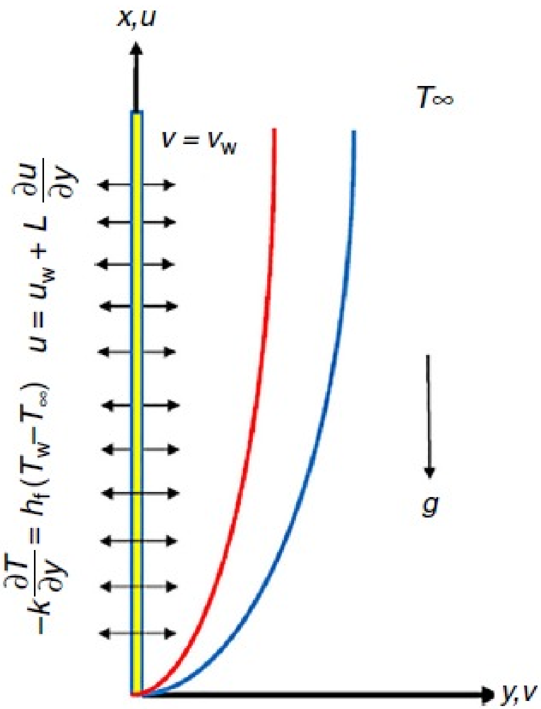

Figure 1
Geometry of the model.

An inviscid, Casson non-Newtonian fluid with electrical conductivity is convecting in a 2D natural turbulent flow through a vertically expanding surface area in the visibility of speed slip. As well as being consistent, the circulation is also laminar. A tendency exists to ignore the impact of induced Magnetic number of Reynolds and the magnetic waves generated in the design process. The effects of temperature radiation, as well as shifting warm sources and sinks, are taken into account. This is a convective limit problem. Fig.1 shows the surface area stretching with a rate $u_w(x) = ax$ towards x. To reverse the circulation instructions, a $B = B_0$ continuous magnetic field is used. You can express Casson fluid's stress-anxiety tension in terms of rheological equations of state, such as

$$\tau_{ij} = 2\left(\mu_\beta + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}\ , \pi > \pi_c 2\left(\mu_\beta + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}\ , \pi < \pi_c \tag{1}$$

here π= $e_{ij}e_{ij}$, e_ij is the part of the contortion rate whereas Casson fluid's plastic vibrant thickness (β) and return tension (π_c) are both components of the contortion rate with each other.

Continuity Equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5.1}$$

Momentum Equation.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu(1 + \frac{1}{\beta})\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2 u}{\rho} + g\rho\beta_T\left(T - T_\infty\right) \tag{5.2}$$

Energy Equation.

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, + q''', \tag{5.3}$$

$$u\frac{\partial C_1}{\partial x} + v\frac{\partial C_1}{\partial y} = D_{S_1}\frac{\partial C_1}{\partial y^2}, \tag{5.4}$$

$$u\frac{\partial C_2}{\partial x} + v\frac{\partial C_2}{\partial y} = D_{S_2}\frac{\partial C_2}{\partial y^2},. \tag{5.5}$$

where v is the viscosity (kinematic), T is the liquid temperature, Is a measure of thermal conductivity, while k is a measure of thermal conductivity; T is the thermal heat capacity, g represents gravity acceleration, q represents liquid thickness, β represents the Casson fluid criterion, c_p represents the warmth capacitance, and q''' represents the variable warm. In this equation, u is the speed component, and v is the viscosity (kinetic). To this idea of a changeable warm energy storage device is better understood, the second term in Equation [3] on the right hand side is provided by

$$q''' = k\frac{u_w}{x\nu}A^*(T_W - T_\infty)f' + B^*(T_W - T_\infty), \tag{5.6}$$

There are two temperatures, $T_W - T_\infty = bx$, whereT_w is the temperature near the edges and T_∞ is the far away temperature. Thermal radiation exchange, according to Rosseland's calculations, includes

$$q_r = \frac{4}{3}\frac{\sigma^*}{k^*}\frac{\partial T^4}{\partial y}. \tag{5.7}$$

where k^* and σ^* are two terminology are used:Stefan Boltzmann constant and mean absorption coefficient (MAC). The Taylor expansion for the series of T^4 around T_∞ is

$$T^4 = 4TT_\infty^3 - 3T_\infty^4 \tag{5.8}$$

where the boundary conditions are as follows: U=$u_w + L\frac{\partial u}{\partial y}$, $v = v_w$, $-k\frac{\partial T}{\partial y} = h_f\,(T_w - T_\infty)$, $C_1 = C_w$, and $C_2 = C_{2w}$ at y=0, u→0, T→T_∞, $C_1 \to C_{1\infty}$, $C_2 \to C_{2\infty}$, as y→∞, here L would be the molecule unrestricted route,$v_w$is the vacuum speed, and h_f is the thermal transfer transmission. As long as $v_w(x) > 0$, we got suction. If v_w (x)< 0, we get an injection.

3. Solution of the Problem

Following are some similarity alterations and non-dimensional variables that can be used to modify Equations. As an example, let's look at the similarity variable η as.

$$\eta = \sqrt{\frac{a}{\nu}}y \tag{5.9}$$

By term of stream function, down below the definition of velocities and the function ψ describe as ψ =$\sqrt{a\nu x}$ f(η) , u=$\frac{\partial\psi}{\partial y}$=ax $f'(\eta)$=, v= -$\frac{\partial\psi}{\partial x}$=$\sqrt{a\nu}$ f(η) as well as liquid velocity and temperature that have no dimensions. In this case, Equations.$(8) and (9)$

satisfy the continuity equation in a trivial way, and thus Equations (2) and (3) become

$$(1+\frac{1}{\beta})f''' + ff'' - f'^2 + Gr\theta + Mf' = 0 \tag{5.10}$$

$$(1+Rd)\theta'' + A^* f' + B^*\theta + Prf\theta' - Prf'\theta = 0 \tag{5.11}$$

$$\phi_1'' + SC_1 f\phi_1' = 0 \tag{5.12}$$

$$\phi_2'' + SC_2 f\phi_2' = 0 \tag{5.13}$$

where primes ($\prime$) refers to differentiation between two variables η where , Grashof no. is Gr $=\frac{g\beta_t D}{a^2}$ and MF parameter M=$\frac{\sigma\beta_0^2}{a\rho}$, Rd=$\frac{16\sigma^* T_\infty^3}{3kk^*}$ is parameter for radiation, Pr = $\frac{\upsilon_\rho c_p}{k}$ is the Prandtl number, S= -$\frac{v_w}{\sqrt{a\upsilon}}$ is the suction injuction parameter and $\gamma = L\sqrt{\frac{a}{\upsilon}}$ is the first order velocity slip parameter (VSP). In this case the boundary conditions are as follows

$f' = 1+\gamma f''$, f= S , $\theta'(0) = -Bi(1-\theta(0))$, $\emptyset_1(\eta) = 1$, $\emptyset_2(\eta) = 1$, at $\eta = 0$

$f'(\eta) \longrightarrow 0$, $\theta(\eta) \longrightarrow 0$, $\emptyset_1(\eta) = 0$, $\emptyset_2(\eta) = 0$ as $\eta \longrightarrow \infty$Where Bi=$\sqrt{\frac{\upsilon}{a}}\frac{h_f}{k}$ is the biot number. The Cf_x(skin friction coefficient) and ν (Nusselt number) are the physical quantities of engineering importance. This is a new form of $Cf_x and \nu$:

$Cf_x\ Re_x^{\frac{1}{2}}$ = (1+$\frac{1}{\beta}$) f$''$(0)

$Nu\ Re_x^{\frac{1}{2}} = -(1+Rd)\,\theta'(0)$ Where R$e_x = \frac{x\mu_w}{\upsilon}$ is the local Reynolds number.

4. Results and Discussion

Equations (10) through (11), which are non-linear ODEs, are numerically fixed using capture and fourth-order Runge-Kuhta techniques in the MATLAB programme. As a result, we can see the impact of non-dimensional controlling criteria. Graphs and tables are used to illustrate Nusselt number. Two and three indicate the influence of magnetic field specification (M) on speed distribution and temperature level area. As M values are increased, the rate field is reduced. According to the Lorentz force, an acceleration in the magnetic field produces in the flow a resistive form of pressure that reduces rate field shape. The variance of temperature is noted to rise with a rise in magnetic criteria. Some extra heat will undoubtedly be produced because of the Lorentz pressure. As the magnetic field increases, the energy layer thins out and the thermal layer thickens. As you can see in Figures $(4) and (5)$, the effectiveness of Casson liquid parameter (β) on rate and temperature level circulations is shown in great detail in these figures. The Casson liquid specification boom, as shown in Fig. 4, has the effect of reducing the circulation rate as well as the matching border layer thickness. An increase in Casson liquid parameter increases the temperature area. As the Casson liquid criteria increases, the thick forces increase as well, and these pressures generate some heat energy in the circulatory system. Due to this, the circulation of heat is greatly accelerated. The radiation criteria (Rd) has a substantial impact on temperature fields, as seen in Figure 6. Radiation parameter increases liquid temperature levels, as seen in Figure 1. Therefore, this power helps in comprehending temperature field by improving the radiation of specification. Thermal zones are represented in $fig.(7) and fig.(8)$ as a result of uneven supplier variables $A^*\&, B^*$. We can see how A^* and B^* are related to heat in plain areas. The schmidt number (Sc), For both active and passive systems, the temperature profile is same. Increasing thermal diffusivity of the nanofluid decreases Prandtl number Pr whilst increasing kinematic viscosity raises Prandtl number Pr, according to our findings in this study. Temperature as well as layer density can be established by increasing the values of irregular warm requirements.$A^* > 0$ and also $B^* > 0$, the fluid is warmed by the presence of this substance. When compared to suction, a higher temperature is obtained with injection. To understand how Biot number affects temperature level region, look at Figure 9. According to the Biot number (10), heat is a rising feature. By increasing the value of, the rate and equivalent energy limit layer density decreased. This may be due to the fact that increasing velocity alters the frictional retardation between the fluid's surface and the fluid's surface. Consequently, the pace of the liquid slowed down. For varied values of rate slip parameter (γ), Fig. 11 shows the temperature level change. Speed slip increases the thickness of the boundary layer. We observed that (γ) has a temperature-increasing characteristic as a result of the dramatic increase in temperature. Temperature and also speed changes are revealed in numbers $12 and 13$. Fig. 12 shows that the speed area increases as Gr increases. Since buoyant and viscous forces are interrelated, Grashof number is a measure of their relationship. As this rate field improves, buoyancy force puts an increasing pressure on the liquid. The decreasing function of Gr can be seen in Fig.

13. Temperature loss is inevitable due to strong buoyant pressure. A^* rise in Gr is associated with an increase in the velocity area as well as a reduction in the thermal field, as a result. Suction/shot parameter (S) has an impact on velocity and temperature, as shown in numbers $14and15$. As a result of the suction/injection parameter, for both pressure and heat zones drop. When suction is applied, the volume of fluid that is attracted to the wall is increased. With larger suction/injection parameters, both energy and thermal boundary layer densities fall. According to numerous physical factors M, Rd, A B Bi, Gr and also S, Considering different values of these factors, we've calculated differences in skin rubbing efficiency and thermal transmission rate. As a result of the table, we can determine the results. The rubbing variable as well as the rate of heat transmission are both reduced when M and are increased. There is an increase in friction factor and In contrast, the local Nusselt number and all other paremeters like β, Rd, A^*, B^*, γ, increases the coefficient of skin rubbing for larger values of Gr and Bi. The schmidt number (Sc), For both active and passive systems, the temperature profile is same. Increasing thermal diffusivity of the nanofluid decreases Prandtl number Pr whilst increasing kinematic viscosity raises Prandtl number Pr, according to our findings in this study. Sc decreases as a result of higher kinematic viscosity and temperature of employed base fluid, and there is no difference between active and passive models in terms of the temperature of the convective fluid at the wall.

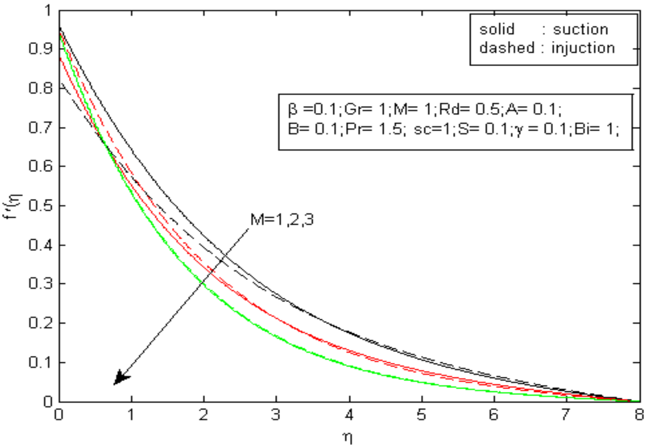

Figure 2: Magnetic field parameter effect on velocity.

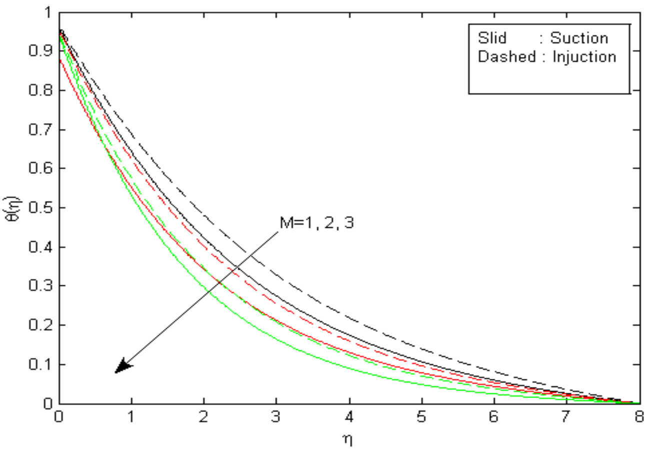

Figure 3: Magnetic field parameter effect on temperature.

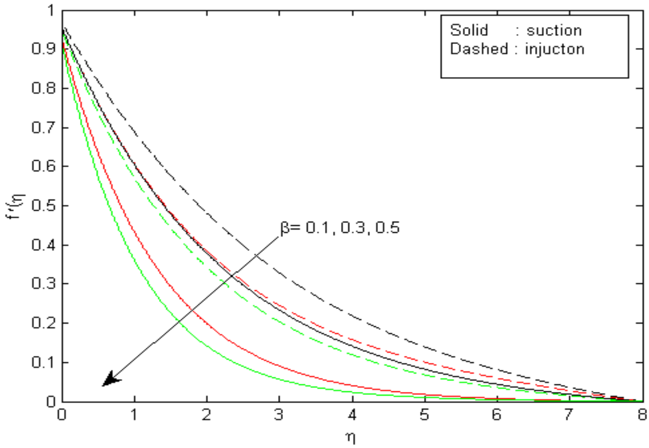

Figure 4: Casson fluid parameter effect on velocity.

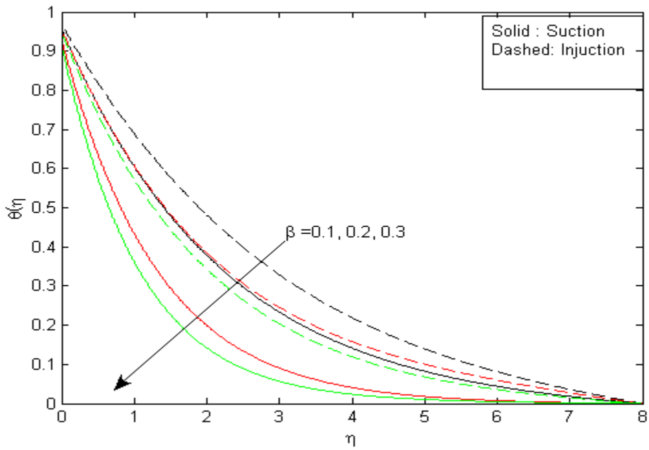

Figure 5: Effect of casson fluid parameter on temperature.

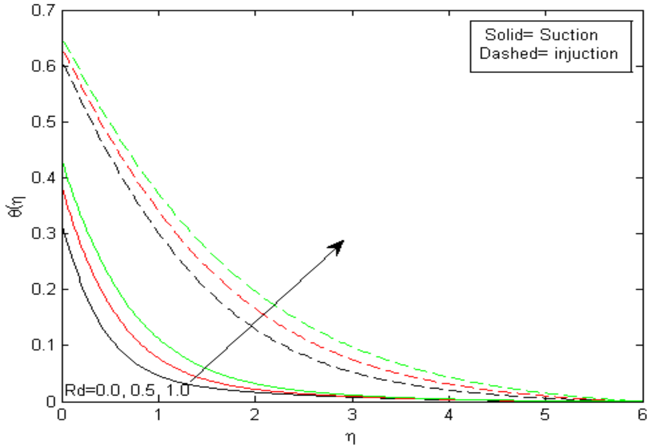

Figure 6: Effect of radiation parametr on temperature.

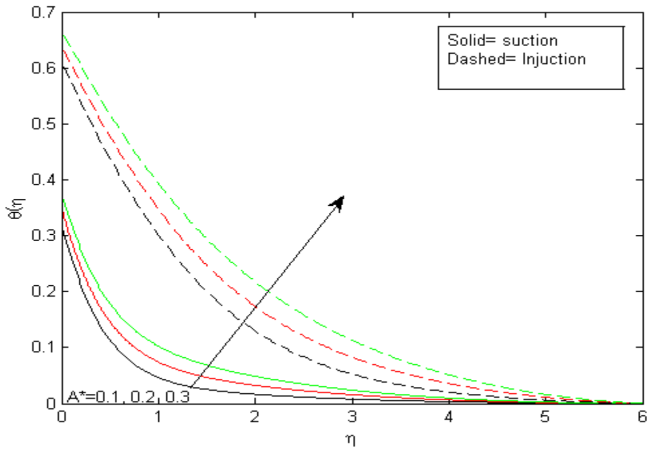

Figure 7: Effect of non uniform A^* on temperature.

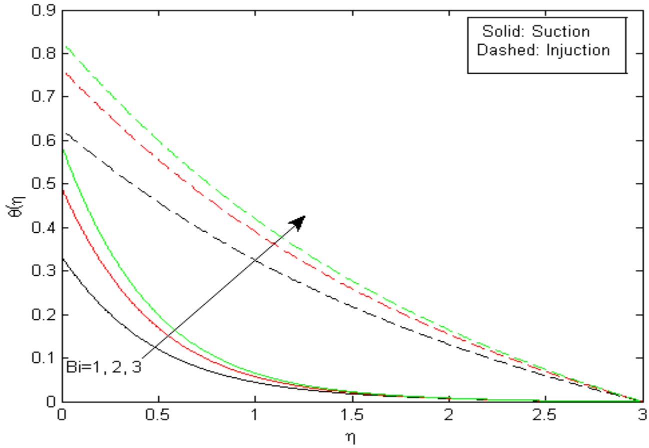

Figure 8: Effect of non uniform parameter B^* on temperature.

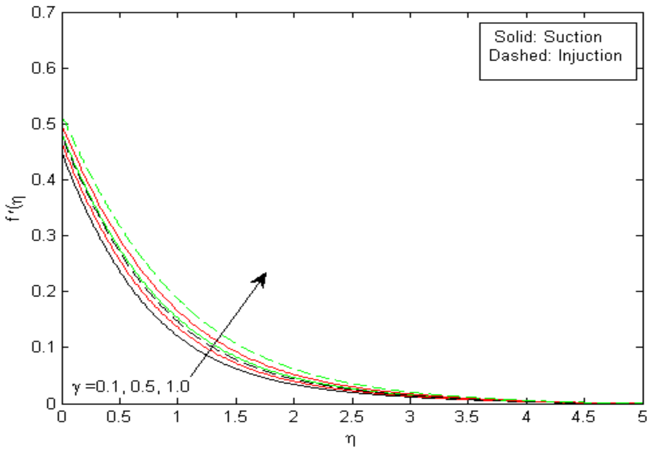

Figure 9: Effect of first order slip parameter on velocity.

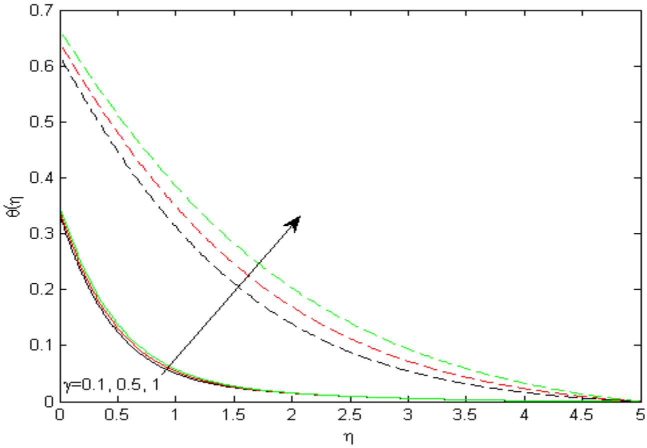

Figure 10: Effect of first order slip parameter on temperature.

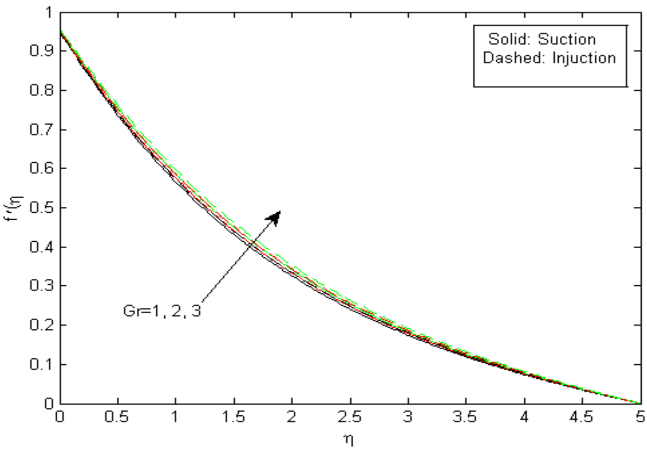

Figure 11: Effect of Grashof number on velocity.

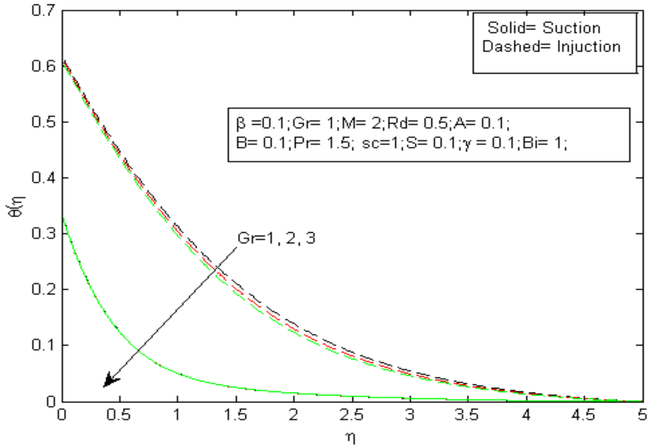

Figure 12: Effect of Grashof number on temperature.

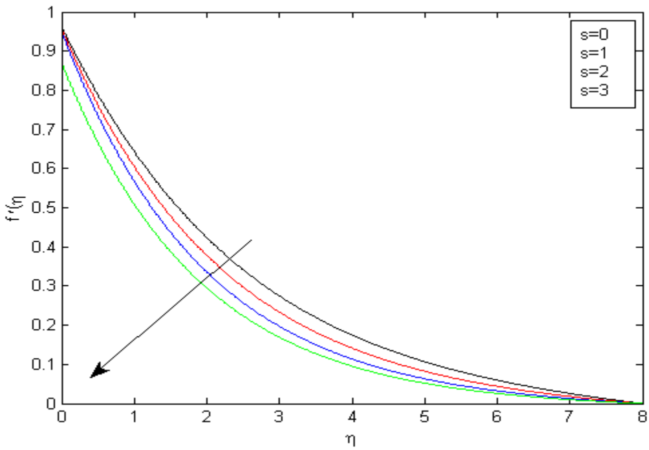

Figure 13: Effect of suction/injuction parameter on velocity.

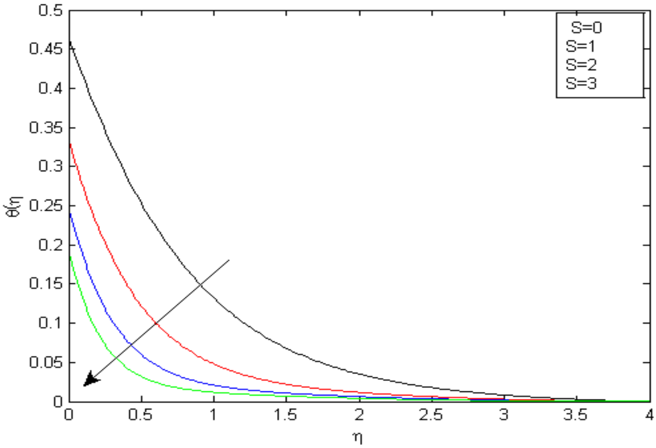

Figure 14: Effect of suction/injuction parameter on temperature.

Conclusion

The mathematical study yields the following conclusions:
(i) The NaCl focus limit layer is thicker than that of sucrose because ions size is smaller in sodium cloride, causing it to diffuse deeper into the liquid than sucrose;
(ii) Increasing the Casson parameter from $\beta = 1.0$ to $\beta = 5.0$ results in an average decrease of 6% in velocity;
(iii) An increase in the mixed convection specification λ leads to a decrease in both the density of the limit layer of velocity and temperature profile, with the effect diminishing as the Prandtl number increases;
(iv) At the wall skin friction reduces the proliferation in Magnetic specification and Casson parameter; and
(v) As convection parameter increases it increases the surface temperature, with 20% increase in heat transfer.

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