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## **Characterizing Transport Phenomena in Non-Newtonian Fluids with Buoyancy and Thermal Slip Effects**

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### **Abstract**

This paper presents research that examines the influence of several parameters, such as MHD, Soret and Dufour effects, buoyancy, and second-order thermal slip, on the flow of Carreau-Yasuda fluid. The underlying boundary layer equations were converted into ordinary differential equations for the investigation, which were then numerically solved for general circumstances using the shooting technique. The results were drawn and thoroughly examined for a variety of parameters, including Brownian parameter, thermophoresis parameter, and magnetic parameter. The findings included velocity, temperature, and concentration patterns within the boundary layer.

### **Nomenclature**

$T_{\infty}$  denotes the ambient fluid temperature

$\nu$  known as kinematic velocity

$\rho$  = density

$v$  = velocity

$d$  = diameter

$\mu$  = viscosity

$C_p$  stand for the constant-pressure specific heat capacity

$\Delta T$  indicate the temperature difference

$C_w$  represent the wall concentration

$C_\infty$  known as the ambient fluid concentration

$C_s$  is the susceptibility of the concentration

## **Introduction**

Non-Newtonian fluids are liquids whose viscosity changes with respect to velocity flow. Non-Newtonian fluids do not follow Newton's viscosity law. There are many fluids in the atmosphere that do not follow the law of viscosity and therefore show their linear relationship between deformation and pressure. Because the change in viscosity occurs with temperature. Just few fluids show such type of steady consistency.

Casson and Carreau-Yasuda non-Newtonian blood viscosity models are basically used to check and analyzed the behavior of two-dimensional Newtonian and non-Newtonian flows for steady and oscillatory flow in straight and curved pipe. In general, this peristaltic phenomenon is very important in medical or biological field. It is most common in the urinary tract, small blood vessels and other glandular ducts in living organism. The importance of peristaltic approach in industry is very clear. We also use this practice in the nuclear industry to obtain toxic fluids. In the past, the peristaltic flow has been extensively tested in direct ways. So in such research biological fluid like blood is considered as viscous fluid. Subsequently attempts were made for peristaltic movement of viscoelastic fluid structures. Viscoelastic fluids cannot be properly described in relation. That's why their skeletal structures are described in a variety of non-Newtonian fluids. But in this kind of fluid, problems arise with their dominant balance of high morality and non-linear behavior. So in solving this problem we will use long wavelength approximation in mathematical modeling. From a variety of non-Newtonian fluids, the Carreau-Yasuda model is more profitable than the so-called power law model. With the help of this model we can estimate the reduction and effect of the cut with great accuracy.

Here, in this paper we analyze the flow of non-Newtonian objects in relation to the effects of Soret and Dufour over a porous surface. There are also the consequences for mixed convection and dissipation. So with this, we first solved non-linear OEEs numerically via shooting method.

The boundary value problem can be solved using the shooting technique by breaking it down into a series of initial value problems. It describes how the resolution of border value issues and initial value problems relate to one another. In the study of differential equation solutions, it is significant from both an academic and practical standpoint. In reality, using this technique, we fire our trajectories in a variety of directions until we discover one with the required boundary value.

Basically, we used just one parameter to find out our missing initial condition. The conditions of boundaries which are not initial, we treat them as a constraints to measure the appropriate values for the parameters. After that we give an initial guess for the parameters to find out the solutions which satisfied the boundary conditions. As we know that every single thing in the universe have advantages as well as disadvantages. Likewise, the shooting method has certain drawbacks or limitations that become evident when the differential equations are unstable, causing them to "blow up" before the initial value problem can be fully integrated. Even highly accurate initial value estimates may fail to prevent this error from occurring. In certain

scenarios, there may exist multiple valid solutions to the boundary value problem, meaning that solving it as an IVP can produce one solution or the other for only a small change in the leftmost boundary condition.

Here in this paper we will use the Shooting method for our non-linear ODEs to solve them numerically. By applying this method when we get solutions compare them with the boundary conditions. After that we need to check the error. We must change the initial approximation of the system and repeat it until the unchanged solution conforms to the conditions of the criteria

Newtonian fluid refers to any fluid that complies with Newton's viscous rule. In Newton's law of viscosity, represents the fluid's dynamic viscosity and the shear tension. Numerous fluids exist that do not adhere to Newton's rule of viscosity; these types of fluids are referred to as non-Newtonian fluids. Because the stress-strain relationship is not followed by all fluids in the universe.

Because of the non-linear connection between deformation rate and tension, most industrial fluids are non-Newtonian in character. These fluids include pulps, blood, molten polymers, and others.

Due to their numerous commercial, biological, and mechanical uses, non-Newtonian fluids have garnered considerable study interest. The connection between shear stress and shear rate for these fluids is nonlinear. The fluid paradigm proposed by Carreau-Yasuda is one such example. Researchers have looked into the flow of Carreau-Yasuda liquid under a variety of flow conditions, including slip effects, curvilinear channels, Hall and Ohmic heating, mixed convection, chemical processes, and radiative flux (Hayat et al., works [11–13]). Peralta et al. [14] examined the Carreau-Yasuda nanofluid flow over a limited vertical sheet in different research. Lee [15] used the weighted least squares method based on finite element analysis to investigate the rheological behavior of the Carreau-Yasuda liquid.

The study of heat transmission has received a lot of interest recently as a result of its numerous uses in the chemical industry, automobiles, microelectronics, and other fields. The abundance of uses for heat transmission in electricity and heating/cooling systems has increased interest among researchers in the field. Choi and Eastman developed a method to accelerate heat transfer by adding nanoparticles to the conventional heat transfer fluids to address the issue of heat transmission. The thermophysical characteristics of a working fluid were examined using nanoparticles with a diameter of 5100nm in the working fluid. We already know that a liquid's heat conductivity depends on the diameter, volume percentage, and bulk of the nanoparticles.

Ahmed et al. [16] Farooq et al. [17] investigated how nanofluids have grown to be very common among modern medical experts, scientists, electronics engineers, mathematicians, and engineers working in the mechanical and material science departments. In general, nanofluids with nm-sized particles have greater stability, higher efficiency, and numerous rheological characteristics. The ambition was sparked by nanoparticles' exceptional

mechanical, thermal, optical, and electronic properties. In a porous material over a plane surface, Khan et al. [18] studied magneto hydrodynamic convection heat transfer flow.

Because magneto-hydrodynamics flow has numerous commercial and engineering uses, it is of paramount significance to study it. By altering the boundary layer structure, the flow field can be directed in a desired direction according to the magneto-hydrodynamics concept. This makes changing the flow dynamics using MHD extremely simple and reliable. MHD has also attracted a lot of interest from physiologists in biology and medicine due to its promise to treat a number of pathological diseases. In their study of boundary layer MHD flow over a nonlinear stretching sheet, Desale et al. [19] came to the conclusion that the magnetic field has a stronger impact on lowering the velocity distribution than the nonlinear expanding parameter Hayat et al. [20] investigated the MHD peristaltic flow of nanofluid in a conduit with slip, wall properties, and Joule heating and discovered that both nanoparticle concentration and temperature are rising functions of thermophoresis and Brownian motion parameters. Akbar et al. [21] studied the numerical solution of MHD Eyring-Powell fluid over an expanding sheet. Mabood et al. [22] investigated the impacts of MHD and viscous loss on a nonlinear stretching sheet. Mahanthesh et al. [23] addressed the findings for both nonlinear and linear sheet-stretching cases.

Due to the Carreau-Yasuda fluid model's capacity to forecast both Newtonian and non-Newtonian behavior, it has been the subject of significant study. On this fluid model, researchers including Hayat et al. [24–26] have carried out a number of experiments. In one research, they looked at the peristaltic flow of Carreau-Yasuda fluid in a curved tube with slip effects and concluded that as the velocity slip parameter is increased, the amount of retrograde pumping and the peristaltic areas drop. In another research, they investigated the peristaltic flow of a viscous, Carreau, and Carreau-Yasuda fluid with Hall and Ohmic heating effects in an uneven channel. A wide wavelength and small Reynolds number estimate were used to study the mixed convective peristaltic movement of the Carreau-Yasuda fluid with chemical reactivity and thermal dissipation. A numerical analysis of the MHD peristaltic transit of Carreau-Yasuda fluid in a curved conduit with Hall effects was also performed by Abbasi et al. [27].

Due to its many uses in areas like engineering, geo-fluid dynamics, and biomechanics, the study of fluid movement through porous surfaces is an extensively studied subject. These fluid movement patterns are also seen and studied in physiological systems found in the human body, including the kidneys, lungs, tiny blood vessels, cartilage, and bones. The movement of blood and nutrition through the body's tissues can be viewed as a deformable porous medium necessary for their correct operation. In order to better understand different illnesses like the development of tumors, researchers have created models to investigate the movement of Newtonian or non-Newtonian fluids through porous surfaces.

Many commercial and environmental processes, particularly those involving cooling and evaporation, depend on the movement of mass and heat. The ability of fluids to transmit heat is essential for comprehending energy processes. It is known as Soret or thermo-diffusion when a temperature gradient results in a mass flow, and Dufour or diffusion-thermo when a concentration gradient results in an energy flux. When examining how heat and mass are transferred, Soret and Dufour effects must be considered concurrently. Additionally, these

impacts are investigated when a heat source or drain is present. References [28-40] contain previous research on Soret and Dufour effects and heat source/sink for different shapes. The goal of the present research is to better understand the coupled heat and mass flux phenomenon with heat source/sink effects that has biological uses.

Chemical engineering and geology both benefit greatly from the Soret effect, which is the diffusion flux brought on by a temperature gradient, and the Dufour effect, which is the heat flux brought on by a chemical potential gradient. Diffusion-thermo effect is essential in mixtures of gases with light and medium molecular weights, and has been used, for instance, to separate isotopes. Coupled heat and mass transmission has many uses in engineering, including the diffusion of medication in blood vessels, the movement of moisture through insulations, and the spread of chemical contaminants. Blood appears to act like a Casson fluid, according to studies [42,43].

Although the effects of Fourier's or Fick's laws are generally regarded to be more significant, diffusion-thermo and thermal-diffusion's effects are sometimes unavoidable. For example, the thermal diffusion phenomenon is utilized in the separation of isotopes as well as in gas mixes with high and middle molecular weights (such as H<sub>2</sub> and He) and air and N<sub>2</sub>, respectively. The purpose of this discussion is to investigate the peristaltic flows of MHD non-Newtonian fluid in a revolving frame while keeping these viewpoints in mind..

A deformable canal with porous sides is a tube that can expand and compress. Uchida and Aoki previously investigated the flow inside a semi-infinite tunnel with contracting/expanding pipelines and impermeable sides. They used an approximate solution to limit the Navier-Stokes equations to a single differential equation, which was then numerically solved to find the major flow features. Afterward, Goto and Uchida investigated a contracting/expanding tunnel with porous walls, while Dauenhauer and Majdalani investigated the movement between parallel contracting/expanding permeable walls. To control the final solution, they examined laminar, unsteady, and incompressible flow inside these walls and added a novel constraint termed injection coefficient. Berman and Majdalani et al. addressed analogous issues, employing physical and wall expansion ratios as factors to convert the general partial differential equations into nonlinear ordinary differential equations. They used perturbation methods to answer these equations analytically before presenting numerical solutions. References [49-58] address recent research into deformable channel issues. Asghar et al. investigated heat transmission in a deformable permeable conduit, analyzing theory and experimental methods.

## Statement of the problem

Consider the Carreau-Yasuda fluid subject to stretched surface. Where the magnetic field is applied in the y-direction with magnitude  $\beta_0$  and the stretching is regarded in the x-direction with stretching rate  $a_1$ .  $T_w$  is assumed to be the surface temperature, and  $C_w$  is assumed to be the percentage. Let  $T_\infty$  and  $C_\infty$  is ambient temperature and concentration.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \Gamma^d \nu \left(\frac{n-1}{d}\right) (d+1) \frac{\partial^2 u}{\partial y^2} \left(\frac{\partial u}{\partial y}\right)^d - \frac{\nu}{\kappa^*} u + g[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] - \frac{\sigma \beta_0^2 u}{\rho}, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_0}{(\rho C_p)} + \frac{\mu_0}{(\rho C_p)} \left(\frac{n-1}{d}\right) \Gamma^d \left(\frac{\partial u}{\partial y}\right)^2 \left(\frac{\partial u}{\partial y}\right)^d + \frac{D_c \kappa_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_c \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_c \kappa_T}{T_m}\right) \frac{\partial^2 T}{\partial y^2}, \tag{4}$$

Conditions on boundary are as follows

$$u - u_w = \beta_v u_y + \beta_v u_{yy}, \quad v = V_0, \quad T - T_w = \beta_t T_y + \beta_t T_{yy}, \quad C = C_w, \quad \text{at } y=0$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{at } y \rightarrow \infty, \tag{5}$$

Where  $\nu$  denotes kinematic viscosity,  $\kappa^*$  porosity rate,  $\kappa$  thermal conductivity,  $\rho$  density,  $C_s$  concentration susceptibility,  $C_p$  specific heat,  $D_c$  mass diffusivity,  $\kappa_T$  thermal mean temperature,  $T_m$  fluid mean temperature,  $\beta_0$  the strength of magnetic field,  $\kappa_r^2$  chemical reaction rate,  $E_a$  activation energy,  $\sigma$  electrical conductivity, and  $\kappa_0$  Boltzman constant.

Transformation is given as;

$$u = axf^{\square}(\eta), \quad v = -\sqrt{av}f(\eta), \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty},$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \eta = y\sqrt{\frac{a}{\nu}}, \tag{6}$$

The governing equation is obtained in its dimensionless form as;

$$f^{\square\square\square} + \left(\frac{n-1}{d}\right) (d+1) f^{\square\square\square} f^{\square\square} (We)^d - \beta^* f^{\square} + \lambda\theta + \lambda N\phi - f^{\square^2} + ff^{\square\square} - M^2 f^{\square} = 0, \tag{7}$$

$$\theta^{\square\square} + PrEcf^{\square^2} \left(1 + \frac{n-1}{d} (We)^d (f^{\square\square})^d\right) + Pr f \theta^{\square} + D_f Pr \phi^{\square} = 0, \tag{8}$$

$$\phi^{\square\square} + LeSr\theta^{\square\square} + Lef\phi^{\square} = 0, \tag{9}$$

The boundary conditions are as follows;

$$f(0) = \frac{-v_0}{\sqrt{av}} = W_0, \quad f'' - d = \gamma_v f'' + f'''' \delta_v, \quad \theta - 1 = \gamma_t \theta'' + \delta_t \theta''',$$

$$\phi(\infty) \rightarrow 0, \quad f''(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(0) = 1,$$

Where  $We$  indicate the weissenberg number,  $\beta^*$  is the porosity number,  $\lambda$  signifies the mixed convection,  $N$  represents the buoyancy ratio,  $Pr$  is the prandtl number,  $Ec$  denotes the Eckert number,  $D_f$  represents the Dufour number,  $Le$  denotes the Lewis number, while  $Sr$  is Soret number,  $M$  is the magnetic field,  $\delta$  is the relative temperature variable,  $\sigma$  the chemical reaction variable. These factors are stand in the form

$$We = \Gamma x \sqrt{\frac{a^3}{v}}, \quad \beta^* = \frac{v}{\kappa^* a}, \quad \lambda = \frac{Gr_x}{Re^2}, \quad Gr_x = \frac{g \beta_T (T_w - T_\infty)}{v^2} x^3, \quad N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)},$$

$$Pr = \frac{(\rho C_p) v}{\kappa}, \quad Ec = \frac{(ax)^2}{c_p (T_w - T_\infty)}, \quad D_f = \frac{D_c \kappa_T (C_w - C_\infty)}{C_s C_p (T_w - T_\infty)}, \quad Le = \frac{v}{D_c}, \quad Re = \frac{ax^2}{v},$$

$$Sr = \frac{D_c \kappa_T (T_w - T_\infty)}{T_m (C_w - C_\infty) v}, \quad \sigma = \frac{\kappa_T^2}{a}, \quad \delta = \frac{T_w - T_\infty}{T_\infty}, \quad M = \sqrt{\frac{\sigma \beta_0^2}{\rho a}},$$

### Mathematical modeling:

Let

$$f = y_1; \quad f'' = y_2; \quad f'''' = y_3; \quad y'''''' = y_3'';$$

$$\theta = y_4; \quad \theta'' = y_5; \quad \theta'''' = y_5'' = y y_2;$$

$$\phi = y_5''; \quad \phi'' = y_7; \quad \phi'''' = y_7'' = y y_3;$$

The boundary conditions are as follows

$$y_1(0) - W_0 = 0, \quad y_2 - d - \gamma y_3 - S r y_3'' = 0, \quad y_2(\infty) \rightarrow 0,$$

$$y_4 - 1 - \gamma_t y_5 - \delta_t y_5'' = 0, \quad y_4(\infty) \rightarrow 0,$$

$$y_6(0) - 1 = 0, \quad y_6(\infty) \rightarrow 0,$$

(10)

$$f = y_1,$$

(11)

$$y_2 = y_1'',$$

(12)

$$y_3 = y_2^{\square},$$

(13)

$$y_3^{\square} = y y_1,$$

(14)

$$\theta = y_4,$$

(15)

$$\phi = y_6,$$

(16)

$$y_3^{\square} + (We)^d \frac{(n-1)(d+1)}{d} y_3^{\square} (y_3)^d - \beta^* y_2 + \lambda y_4 + N \lambda y_6 - y_2^2 + y_1 y_3 - M y_2 = 0$$

$$y_3^{\square} \left[ 1 + (We)^d \frac{(n-1)(d+1)}{d} (y_3)^d \right] = \beta^* y_2 - \lambda y_4 - N \lambda y_6 + y_2^2 - y_1 y_3 + M y_2$$

$$y_3^{\square} = \frac{\beta^* y_2 - \lambda y_4 - N \lambda y_6 + y_2^2 - y_1 y_3 + M y_2}{1 + (We)^d \frac{(n-1)(d+1)}{d} (y_3)^d}$$

(17)

$$\theta^{\square} = y_5,$$

(18)

$$\theta^{\square\square} = y_5^{\square},$$

(19)

$$\phi^{\square} = y_7,$$

(20)

$$\phi^{\square\square} = y_7^{\square} = y y_3,$$

(21)

$$y_5^{\square} = -Pr Ec y_3^2 \left[ 1 + \frac{(n-1)}{d} (We)^d y_3^d \right] - Pr y_1 y_5 - D_f Pr y y_3$$

(22)

$$\phi^{\square\square} + Le Sr \theta^{\square\square} + Le f \phi^{\square} = 0$$

(23)

$$y_7^{\square} = -Le Sr y_5^{\square} - Le y_1 y_7$$

(24)

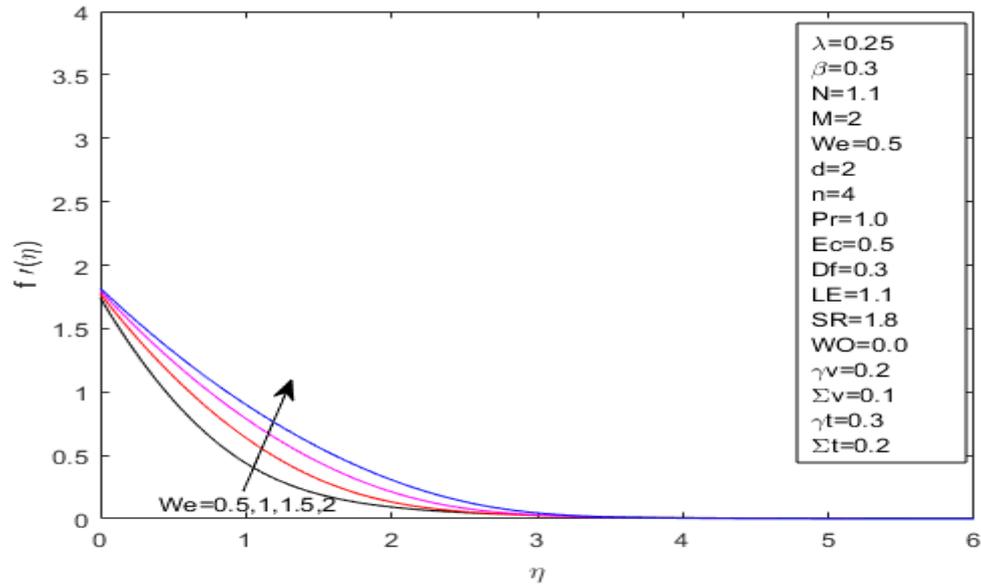


Fig.1

In this section we analysis the behavior of velocity profiles with respect to various variables. Clearly, in the Fig 5.1 velocity rises as increase occur in the  $We$ . As we know that the relation of  $We$  with viscosity is inversely proportional. So if liquid is less viscous then surely it has more velocity.

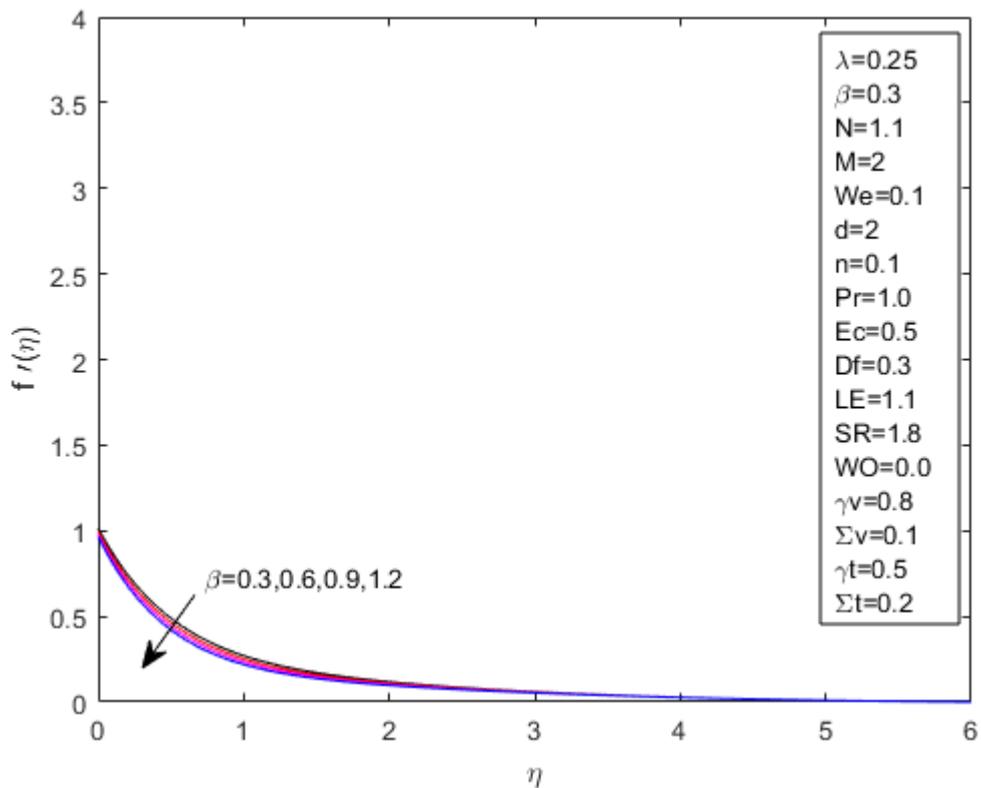


Fig 2

In fig. 2 clearly seen that there is decrease in velocity as increase in the porosity parameter. Actually when we increase the value of porosity variable there is more resistance occurred in the working of fluid or liquid. Therefore this graph shows decline in velocity of liquid.

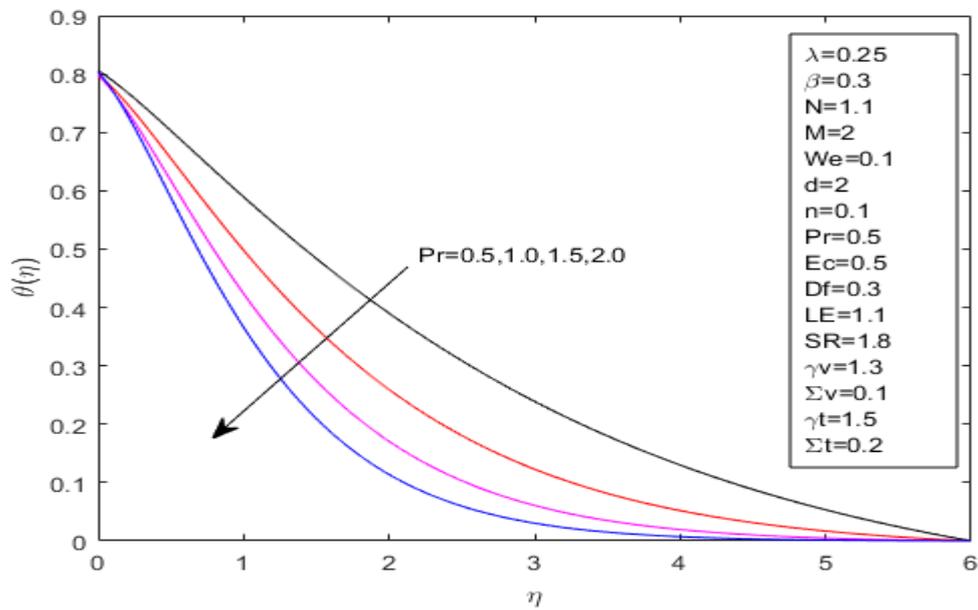


Fig 3

In fig. 3 we examined the impact of temperature under various flow variable. It is obvious that raising the Prandtl value reduces warmth. It occurs as a result of a decline in temperature diffusivity. Physically, the inside operation of the fluid, which generates heat, increased the rate of diffusion.

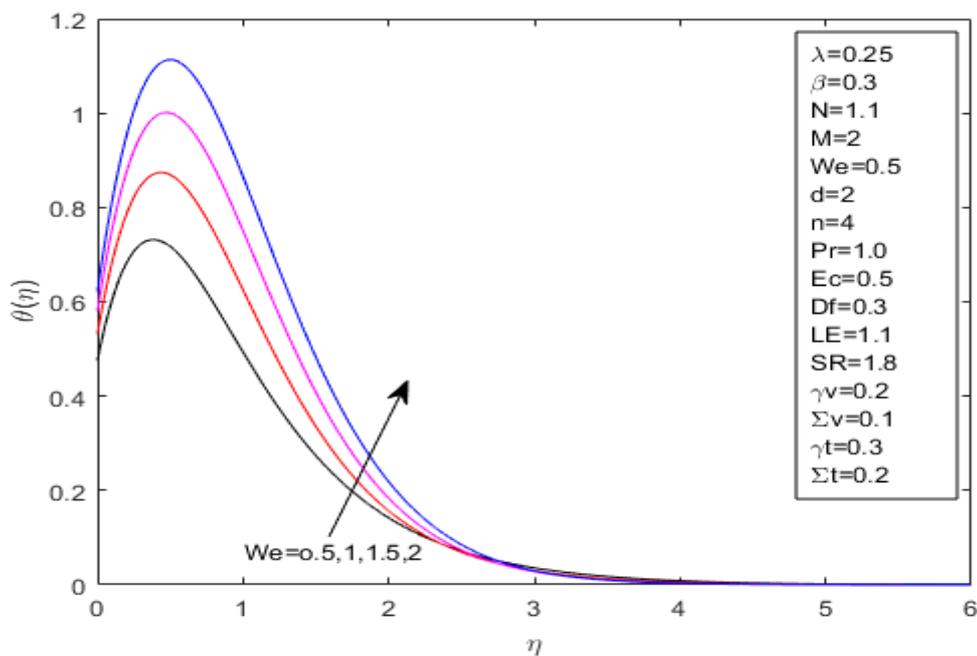


Fig 4

Due to an increase in moving energy, raising the We number raises temperature. In Fig. 4, it was shown that temperature rises as We number rises. When the We number is raised, thermal diffusivity is seen, which raises the temperature.

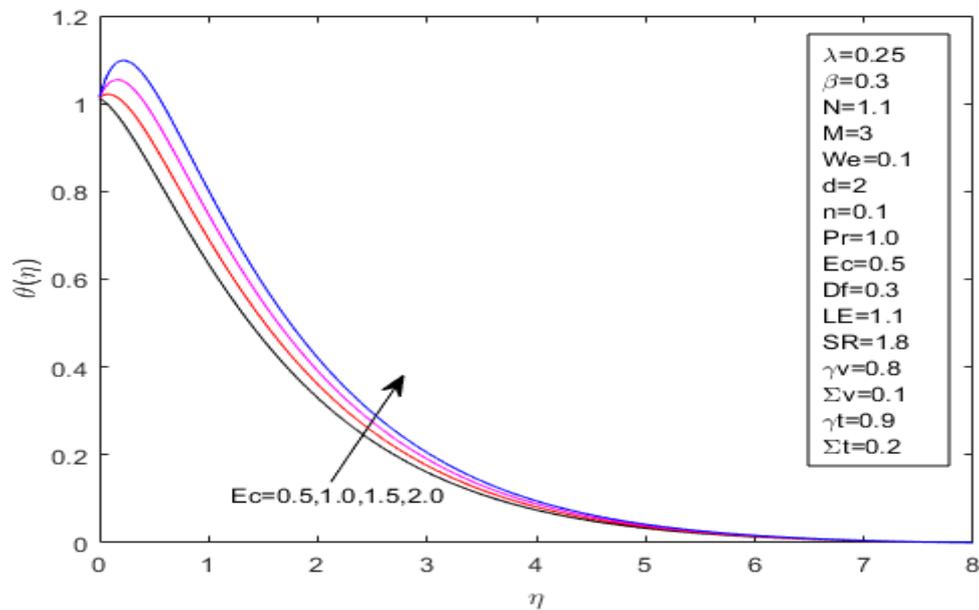


Fig 5

Figure 5 analyzes the relationship between temperature behavior and Ec number. By raising the Ec number owing to an increase in rotational energy, temperature is improved. The system's kinetic energy increases with greater values than the Eckert number, leading to a rise in temperature.

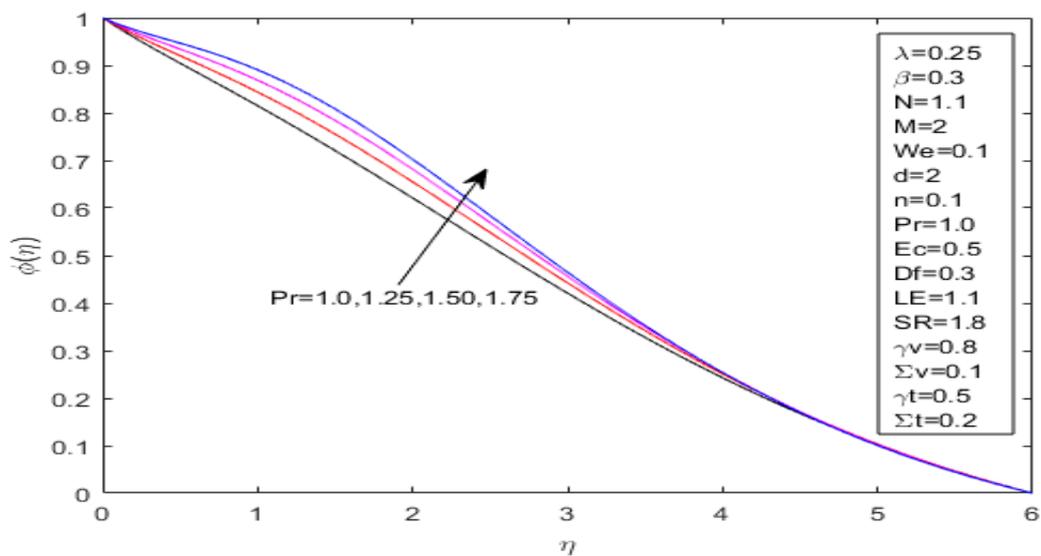


Fig 6

Fig. 6 influence the concentration profile with varying flow variable which is Prandtl number. As we seen that concentration profile is increasing with the higher values of Prandtl number.

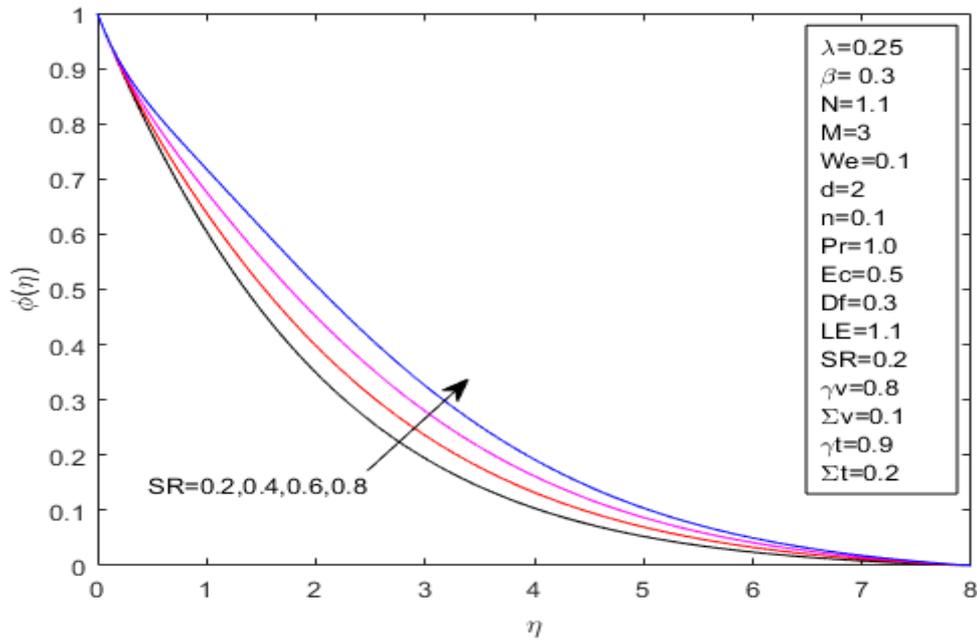


Fig 7

Clearly, Fig.7 depicts that increase in concentration with the higher values of Soret number. With increase in the value of Soret number concentration also enhanced.

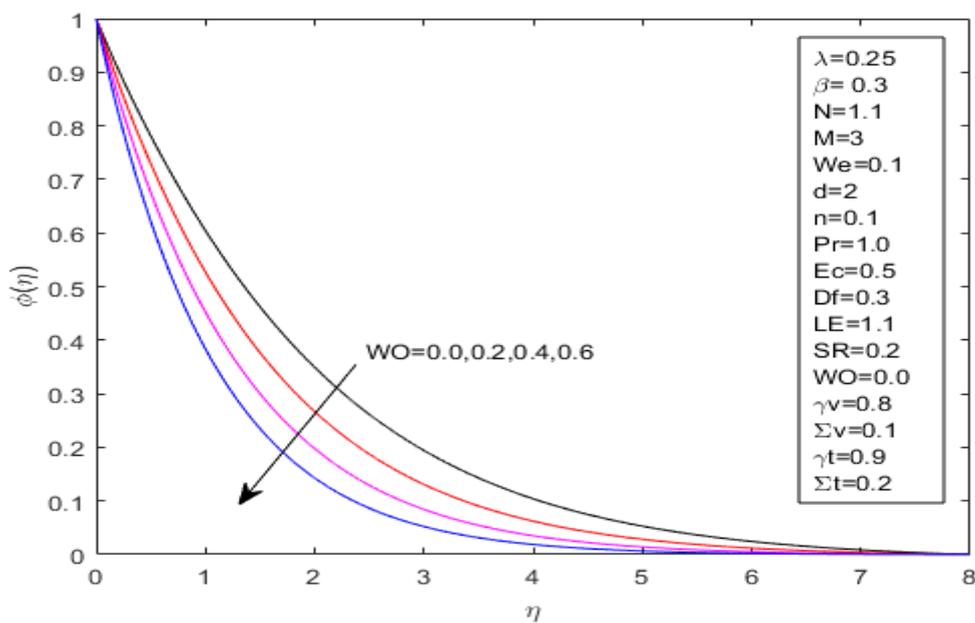


Fig 8

From the Fig. 8 we illustrate that with the higher values of suction parameters there is a decrement in the concentration of the fluid.

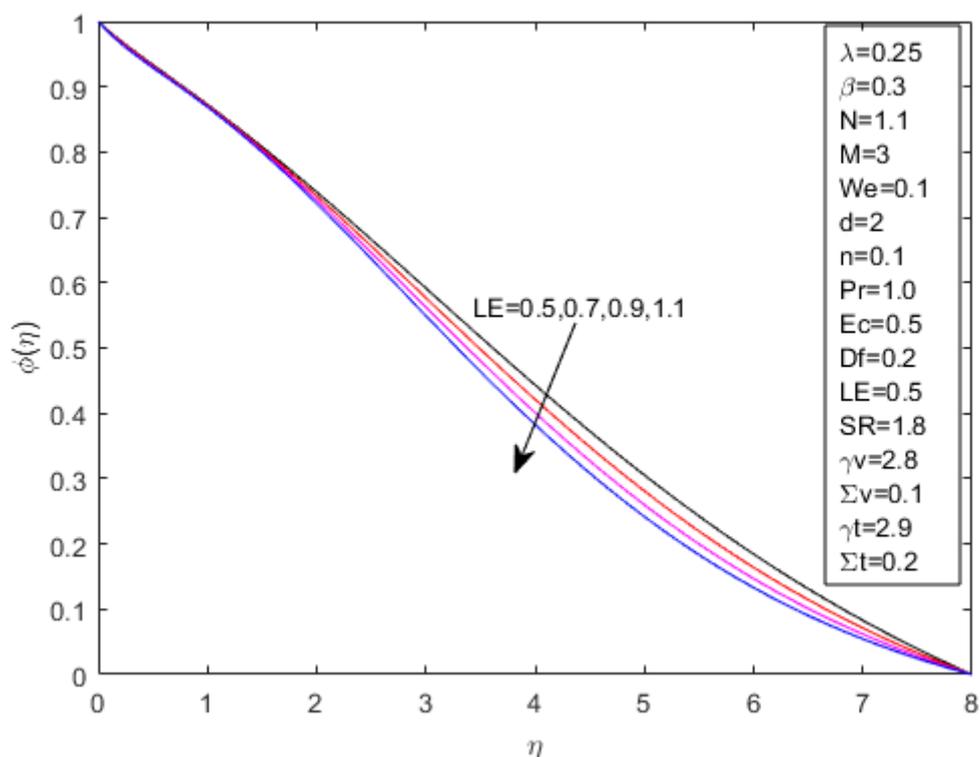


Fig 9

Fig. 9 shows that concentration is decline. Clearly, with the increment in LE number there is decrement in the concentration of fluid.

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